# Asymmetric Volatility and Risk in Equity Markets

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It appears that volatility in equity markets is asymmetric: returns and conditional volatility are negatively correlated. We provide a unified framework to simultaneously investigate asymmetric volatility at the firm and the market level and to examine two potential explanations of the asymmetry: leverage effects and volatility feedback. Our empirical application uses the market portfolio and portfolios with different leverage constructed from Nikkei 225 stocks. We reject the pure leverage model of Christie (1982) and find support for a volatility feedback story. Volatility feedback at the firm level is enhanced by strong asymmetries in conditional covariances. Conditional betas do not show significant asymmetries. We document the risk premium implications of these findings.

There is a long tradition in finance [see, e.g., Cox and Ross (1976)] that models stock return volatility as negatively correlated with stock returns. Influential articles by Black (1976) and Christie (1982) further document and attempt to explain the asymmetric volatility property of individual stock returns in the United States. The explanation put forward in these articles is based on leverage. A drop in the value of the stock (negative return) increases financial leverage, which makes the stock riskier and increases its volatility.[1]

Although, to many, "leverage effects" have become synonymous with asymmetric volatility, the asymmetric nature of the volatility response to return shocks could simply reflect the existence of time-varying risk premiums [Pindyck (1984), French, Schwert, and Stambaugh, (1987), and Campbell and Hentschel (1992)]. If volatility is priced, an anticipated increase in volatility raises the required return on equity, leading to an

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[1] Black (1976) also discusses an operating leverage effect, induced by fixed costs of the firm, but that effect has received little attention in the finance literature.

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immediate stock price decline. Hence the causality is different: the leverage hypothesis claims that return shocks lead to changes in conditional volatility, whereas the time-varying risk premium theory contends that return shocks are caused by changes in conditional volatility.

Which effect is the main determinant of asymmetric volatility remains an open question. Studies focusing on the leverage hypothesis, such as Christie (1982) and Schwert (1989), typically conclude that it cannot account for the full volatility responses. Likewise, the time-varying risk premium theory enjoys only partial success. The volatility feedback story relies first of all on the well-documented fact that volatility is persistent. That is, a large realization of news, positive or negative, increases both current and future volatility. The second basic tenet of this theory is that there exists a positive intertemporal relation between expected return and conditional variance. The increased volatility then raises expected returns and lowers current stock prices, dampening volatility in the case of good news and increasing volatility in the case of bad news. Whereas such a relationship for the market portfolio would be consistent with the capital asset pricing model [CAPM; Sharpe (1964)], it only holds in general equilibrium settings under restrictive assumptions [see Backus and Gregory (1993), Campbell (1993), and the discussion in Glosten, Jagannathan, and Runkle (1993)].

Moreover, there are conflicting empirical findings. For example, French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1992) find the relation between volatility and expected return to be positive, while Turner, Startz, and Nelson (1989), Glosten, Jagannathan, and Runkle (1993), and Nelson (1991) find the relation to be negative. Often the coefficient linking volatility to returns is statistically insignificant. If the relation between market conditional volatility and market expected return is not positive, then the validity of the time-varying risk premium story is in doubt.

Furthermore, the time-varying risk premium story does not readily explain the existence of volatility asymmetry at the firm level, since, in the CAPM example, the relevant measure of risk is then the covariance with the market portfolio. For the time-varying risk premium story to explain firm-specific volatility asymmetry, covariances with the market portfolio should respond positively to increases in market volatility.

Our first contribution is to develop a general empirical framework to examine volatility asymmetry at the market level and at the firm or portfolio level simultaneously and to differentiate between the two competing explanations. That such an analysis has not been done before reflects the existence of two virtually separate literatures. As the survey of empirical articles in Table 1 shows, studies focusing on the time-varying risk premium story typically use market-level returns, whereas

**Table 1**
**Summary of selected empirical studies on asymmetric volatility**

| Study | Volatility measure | Presence of asymmetry | Explanation |
|---|---|---|---|
| Black (1976) | Gross volatility | Stocks, portfolios | Leverage hypothesis |
| Christie (1982) | Gross volatility | Stocks, portfolios | Leverage hypothesis |
| French, Schwert and Stambaugh (1987) | Conditional volatility | Index | Time-varying risk premium theory |
| Schwert (1990) | Conditional volatility | Index | Leverage hypothesis |
| Nelson (1991) | Conditional volatility | Index | Unspecified |
| Campbell and Hentschel (1992) | Conditional volatility | Index | Time-varying risk premium theory |
| Cheung and Ng (1992) | Conditional volatility | Stocks | Unspecified |
| Engle and Ng (1993) | Conditional volatility | Index (Japan Topix) | Unspecified |
| Glosten, Jagannathan and Runkle (1993) | Conditional volatility | Index | Unspecified |
| Bae and Karolyi (1994) | Conditional volatility | Index | Unspecified |
| Braun, Nelson and Sunier (1995) | Conditional volatility | Index and stocks | Unspecified |
| Duffee (1995) | Gross volatility | Stocks | Leverage hypothesis |
| Ng (1996) | Conditional volatility | Index | Unspecified |
| Bekaert and Harvey (1997) | Conditional volatility | Index (Emerging Markets) | Unspecified |

This table lists a sample of studies on the relationship between returns and conditional volatility. Conditional volatility studies typically use GARCH models to measure volatility; "gross volatility" typically refers to the standard deviation of daily returns computed over the course of a month. The "unspecified" label in the explanation column means that asymmetry was modeled but the researchers did not specify the exact cause of asymmetry.

studies focusing on the leverage hypothesis typically use firm or portfolio data. Moreover, the empirical specifications are not entirely compatible across the two literatures. Studies focusing on individual firms typically use regression analysis to examine the relation between a measure of volatility during a particular month ("gross" volatility) and the return in the previous month. Studies at the market level have mostly used the GARCH-in-mean framework of Engle, Lilien, and Robbins (1987), focusing on the relation between return innovations and the conditional volatility of the returns (see Table 1). Our model, while using a related framework, nests the riskless debt model for individual firms in Christie (1982).

Our second contribution is to document a new phenomenon that helps explain volatility asymmetry at the firm level: covariance asymmetry. When the conditional covariance between market and stock returns responds more to negative than to positive market shocks the volatility feedback effect is particularly strong. Our empirical framework accommodates this possibility and we find evidence of such covariance asymmetry. Although Kroner and Ng (1998) document covariance asymmetry in the volatility dynamics of portfolios of small and large firms, most previous studies have focused on asymmetric effects in conditional betas [see Ball and Kothari (1989), Braun, Nelson, and Sunier (1995)] with conflicting empirical results. We argue below that asymmetry is more likely to be found in conditional covariances and re-examine whether conditional betas display asymmetry for our sample.

Third, since our model combines modeling volatility dynamics and risk premiums, we quantify the risk implications of the estimated volatility dynamics. Most applications of GARCH models, with a few exceptions, have not yet embraced asymmetric volatility models. For example, parameterizations of CAPM models that use GARCH [see, e.g., Engel et al. (1995)], models of volatility spillover across equity markets [see, e.g., Hamao, Masulis, and Ng (1990)], and stochastic volatility models for options [Hull and White (1987)] have typically not used asymmetric volatility models.[2] This is surprising since a number of sophisticated models have been developed to accommodate asymmetric volatility [see, e.g., Nelson (1991), Glosten, Jagannathan, and Runkle (1993), and Hentschel (1995)], and the results in Pagan and Schwert (1990) and Engle and Ng (1993) indicate that these volatility models outperform standard GARCH models. If these models yield different conditional volatilities from symmetric GARCH models, their economic implications will be different too. With an asymmetric volatility model, risk and the cost of capital may increase more in response to negative market return shocks than in response to positive shocks. Whereas the economic importance of such effects is indisputable, it is not ex ante clear that statistically significant asymmetric volatility has economically important risk implications.

Finally, whereas most of the empirical analysis so far (see Table 1) has focused on U.S. stock returns, our empirical application focuses on the market return and portfolio returns constructed from Japanese stocks in the Nikkei index. As Engle and Ng (1993) conclude for the Japanese Topix index, our results indicate that asymmetry is an important feature of stock market volatility in the Japanese market as well.

The remainder of the article is organized as follows. Section 1 formulates our empirical model, the empirical hypotheses, and explains the role of leverage in generating asymmetric risk and volatility. A set of specification tests is also discussed. Section 2 discusses the data and the empirical results. Section 3 considers the economic implications of our model and Section 4 evaluates the robustness of the empirical results. The final section summarizes the results and outlines directions for further research.

[2] Exceptions are Koutmos and Booth (1995) and Ng (1996) in the volatility spillover literature and Amin and Ng (1993), Duan (1995), and Wu (1998) in the options literature.

## 1. A Model of Asymmetric Volatility and Risk

### 1.1 Asymmetric volatility and risk at the firm and market level

To establish notation, let $P_{M,t}$ denote the market index, let $r_{M,t}$ denote the return on the market portfolio, and let $r_{M,t+1} = E(r_{M,t+1}|I_t) + \epsilon_{M,t+1}$, where $I_t$ denotes the information set at time $t$. Similarly, $P_{i,t}, r_{i,t}$ are the price and return of the stock of firm $i$, respectively, and $r_{i,t+1} = E(r_{i,t+1}|I_t) + \epsilon_{i,t+1}$. Define conditional variances and covariances, $\sigma^2_{M,t+1} = \text{var}(r_{M,t+1}|I_t)$, $\sigma^2_{i,t+1} = \text{var}(r_{i,\ t+1}|I_t)$ and $\sigma_{iM,t+1} = \text{cov}(r_{i,t+1}, r_{M,t+1}|I_t)$.

***Definition.*** *A return* $r_{i,t}$ *displays asymmetric volatility* if

$$\text{var}\left[r_{i,t+1}|I_t, \epsilon_{i,t} < 0\right] - \sigma^2_{i,t} > \text{var}\left[r_{i,t+1}|I_t, \epsilon_{i,t} > 0\right] - \sigma^2_{i,t}. \quad (1)$$

In other words, negative unanticipated returns result in an upward revision of the conditional volatility, whereas positive unanticipated returns result in a smaller upward or even a downward revision of the conditional volatility.[3]

One explanation for such asymmetry at the equity level relies on changes in leverage. To illustrate, consider a world where debt is riskless, that is, the return on all debt equals the risk-free rate. We denote the risk-free rate by $r^f_{t-1,t}$, since it is known at $t-1$. It is straightforward to show that

$$r_{i,t} - r^f_{t-1,t} = (1 + LR_{i,t-1})\left(\bar{r}_{i,t} - r^f_{t-1,t}\right), \quad (2)$$

where $LR_{i,t-1}$ is the leverage ratio for firm $i$ and $\bar{r}_{i,t}$ refers to the return on the firm's assets.[4] Even when the volatility of the return on a firm's assets is constant, the conditional volatility of the equity return should change when leverage changes [see also Christie (1982) and Schwert (1989)]. In particular, shocks that increase the value of the firm, reduce leverage, and with it the conditional volatility of the stock's return and vice versa.

[3] We will refer to the latter case as "strong asymmetry," which implies

$$\text{var}\left[r_{t+1}|I_t, \epsilon_t > 0\right] - \sigma_t^2 < 0, \quad \textit{and}$$
$$\text{var}\left[r_{t+1}|I_t, \epsilon_t < 0\right] - \sigma_t^2 > 0.$$

[4] With $D_{i,t}(E_{i,t})$ denoting the value of debt (equity), the leverage ratio is the debt:equity ratio: $LR_{i,t} \equiv D_{i,t}/E_{i,t}$. The firm return is the value-weighted sum of the return on debt and the return on equity, $\bar{r}_{i,t} = \frac{D_{i,t-1}}{D_{i,t-1}+E_{i,t-1}} r^f_{t-1,t} + \frac{E_{i,t-1}}{D_{i,t-1}+E_{i,t-1}} r_{i,t}$. Multiplying both sides by $D_{i,t-1} + E_{i,t-1}$ and dividing through by $E_{i,t-1}$, we obtain Equation (2) after rearranging terms.

Our analysis here is premised on two assumptions, which we test below. First, we assume that a conditional version of the CAPM holds, that is, the market portfolio's expected excess return is the (constant) price of risk times the conditional variance of the market and the expected excess return on any firm is the price of risk times the conditional covariance between the firm's return and the market. Note that we formulate the volatility feedback effect at the level of the firm's total assets, since it does not at all depend on leverage. Second, we assume that conditional volatility is persistent, which is an empirical fact supported by extensive empirical work [see Bollerslev, Chou, and Kroner (1992)]. Since the time variation in second moments is not restricted by the CAPM, we explicitly parameterize it in the next subsection. For now, we consider more generally the mechanisms generating asymmetry, including leverage and volatility feedback, at the market level and firm level using the flow chart in Figure 1.

We begin by considering news (shocks) at the market level. Bad news at the market level has two effects. First, whereas news is evidence of higher current volatility in the market, investors also likely revise the conditional variance since volatility is persistent. According to the

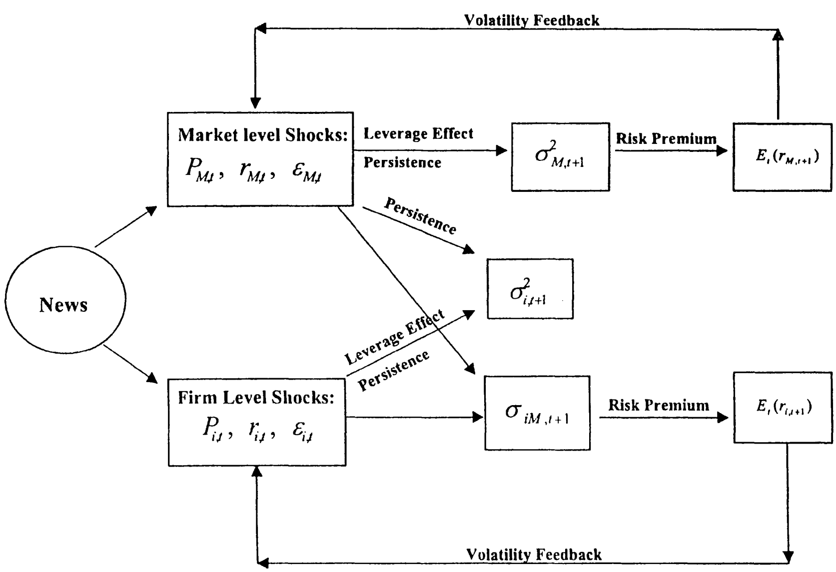


**Figure 1**
**News impact at the market level and the firm level**
This figure shows the impact of market ($\epsilon_{M,t}$) and firm ($\epsilon_{i,t}$) shocks on conditional variances ($\sigma^2_{M,t+1}, \sigma^2_{i,t+1}$) and covariances ($\sigma_{iM,t+1}$). Feedback effects on current prices ($P_{i,t}, P_{M,t}$) and returns ($r_{i,t}, r_{M,t}$) originating from risk premium changes are also shown.

CAPM, this increased conditional volatility at the market level has to be compensated by a higher expected return, leading to an immediate decline in the current value of the market [see also Campbell and Hentschel (1992)]. The price decline will not cease until the expected return is sufficiently high. Hence a negative return shock may generate a significant increase in conditional volatility. Second, the marketwide price decline leads to higher leverage at the market level and hence higher stock volatility. That is, the leverage effect reinforces the volatility feedback effect. Note that although the arrows in Figure 1 suggest a sequence of events, the effects described above happen simultaneously, that is, leverage and feedback effects interact.

When good news arrives in the market, there are again two effects. First, news brings about higher current period market volatility and an upward revision of the conditional volatility. When volatility increases, prices decline to induce higher expected returns, offsetting the initial price movement. The volatility feedback effect dampens the original volatility response. Second, the resulting market rally (positive return shock) reduces leverage and decreases conditional volatility at the market level. Hence the net impact on stock return volatility is not clear.

As Figure 1 shows, for the initial impact of news at the firm level, the reasoning remains largely the same: bad and good news generate opposing leverage effects which reinforce (offset) the volatility embedded in the bad (good) news event. What is different is the volatility feedback. A necessary condition for volatility feedback to be observed at the firm level is that the covariance of the firm's return increases in response to market shocks. If the shock is completely idiosyncratic, the covariance between the market return and individual firm return should not change, and no change in the required risk premium occurs. Hence idiosyncratic shocks generate volatility asymmetry purely through a leverage effect. Volatility feedback at the firm level occurs when marketwide shocks increase the covariance of the firm's return with the market. Such covariance behavior would be implied by a CAPM model with constant (positive) firm betas and seems generally plausible. The impact on the conditional covariance is likely to be different across firms. For firms with high systematic risk, marketwide shocks may significantly increase their conditional covariance with the market. The resulting higher required return then leads to a volatility feedback effect on the conditional volatility, which would be absent or weaker for firms less sensitive to market level shocks. From Equation (2), it also follows that any volatility feedback effect at the firm level leads to more pronounced feedback effects at the stock level the more leveraged the firm is.

The volatility feedback effect would be stronger if covariances respond asymmetrically to market shocks. We call this phenomenon *covariance asymmetry*. So far, covariance asymmetry has primarily received attention in the literature on international stock market linkages, where larger comovements of equity returns in down markets adversely affect the benefits of international diversification [Ang and Bekaert (1998) and Das and Uppal (1996)]. Kroner and Ng (1998) document covariance asymmetry in stock returns on U.S. portfolios of small and large firms without providing an explanation.

There are two channels through which covariance asymmetry can arise naturally and both channels are embedded in our empirical specification. First, covariance asymmetry in stock returns could be partially explained by a pure leverage effect, without volatility feedback. Using the riskless debt model, it follows that

$$\begin{aligned}\operatorname{cov}_{t-1}&\left[r_{i,t}-r^f_{t-1,t}, r_{M,t}-r^f_{t-1,t}\right]\\&=(1+LR_{i,t-1})(1+LR_{M,t-1})\\&\quad\times\operatorname{cov}_{t-1}\left[\bar{r}_{i,t}-r^f_{t-1,t}, \bar{r}_{M,t}-r^f_{t-1,t}\right].\end{aligned} \tag{3}$$

Even with constant covariance at the firm level, the covariance of an individual stock return with the market may exhibit (strong) asymmetry. Conditional stock return betas are somewhat less likely to display pure leverage effects, since

$$\beta_{i,t-1}=\frac{1+LR_{i,t-1}}{1+LR_{M,t-1}}\bar{\beta}_{i,t-1}, \tag{4}$$

where $\bar{\beta}_{i,t-1}(\beta_{i,t-1})$ is the firm (stock) beta. Hence, idiosyncratic shocks should result in asymmetric beta behavior, but the effect of marketwide shocks on betas is ambiguous.

Second, at the firm level as well, covariance asymmetry arises more naturally than beta asymmetry. Suppose the conditional beta of a firm is positive but constant over time, still a popular assumption in many asset pricing models. Then the conditional covariance with the market return is proportional to the conditional variance of the market. Hence a market shock that raises the market's conditional variance increases the required risk premium on the firm (unless the price of risk changes) and causes a volatility feedback effect. When the effect of the market shock on market volatility is asymmetric, the firm (and stock) return automatically displays covariance asymmetry. Of course, betas do vary over time [see Jagannathan and Wang (1995) and Ghysels (1998) for recent discussions] and may exhibit asymmetry as well, but there is no model we know of that predicts beta asymmetry at the firm level. In the framework set out below, we impose only mild restrictions on the behavior of betas over time and we examine whether they exhibit asymmetry.

## 1.2 Empirical model specification

We use a conditional version of the CAPM to examine the interaction between the means and variances of individual stock returns and the market return. The conditional mean equations are defined as

$$\begin{cases} r_{M,t} - r^f_{t-1,t} = Y_{t-1}\sigma^2_{M,t} + \epsilon_{M,t} \\ r_{1,t} - r^f_{t-1,t} = Y_{t-1}\sigma_{1M,t} + \epsilon_{1,t} \\ \vdots \\ \vdots \\ r_{n,t} - r^f_{t-1,t} = Y_{t-1}\sigma_{nM,t} + \epsilon_{n,t} \end{cases}, \tag{5}$$

where $r^f_{t-1,t}$ is the one-period risk-free interest rate known at time $t-1, Y_{t-1}$ is the price of risk, $M$ denotes the market portfolio, and $n$ is the number of other portfolios included in the study. Naturally these portfolios are classified by the leverage ratios of the underlying firms, with portfolio 1 having the highest leverage and portfolio $n$ the lowest. We call these portfolios the leverage portfolios.

The time variation in the price of risk depends on market leverage:

$$Y_{t-1} = \frac{Y}{1 + LR_{M,t-1}}. \tag{6}$$

This specification for the price of risk follows from formulating the CAPM at the firm level, not the equity level, with a constant price of risk. That is,

$$Y = \frac{E_{t-1}[\bar{r}_{M,t}] - r^f_{t-1,t}}{\bar{\sigma}^2_{M,t}}, \tag{7}$$

where the bars indicate firm values rather than equity values. Under certain assumptions, $Y$ is the aggregate coefficient of relative risk aversion [see Campbell (1993)]. It is critical in this context that the return used in Equation (7) is a good proxy to the return on the aggregate wealth portfolio. Since the stock index we use in the empirical work is highly levered, $\bar{r}_{M,t}$ is a better proxy[5] than $r_{M,t}$. Of course, the specification in Equation (6) relies on the riskless debt model. However, we subject the model to a battery of specification tests, some of which are specifically designed with alternatives to the riskless debt model in mind.

[5] Jagannathan, Kubota, and Takehara (1998) argue that a portfolio of listed stocks is unlikely to be a good proxy for the aggregate wealth portfolio in Japan and find that labor income is priced. They ignore leverage effects, however.

Since the CAPM does not restrict the time variation in second moments, we employ a multivariate GARCH model. Specifically, the variance-covariance matrix follows an asymmetric version of the BEKK model [Baba et al. (1989), Engle and Kroner (1995), and Kroner and Ng (1998)]. This GARCH-in-mean parameterization of the CAPM, incorporating an equation for the market portfolio, is similar to the international CAPM parameterization in Bekaert and Harvey (1995) and DeSantis and Gerard (1997), with more general volatility dynamics. In particular, note that the individual shocks need not add up to the market portfolio shock, since we only consider a limited number of leverage-sorted portfolios.

To clearly distinguish the leverage effect from volatility feedback, we formulate our GARCH model at the *firm* level.

Define

$$\bar{\epsilon}_t = \begin{pmatrix} \bar{\epsilon}_{M,t} \\ \bar{\epsilon}_{1,t} \\ \vdots \\ \bar{\epsilon}_{n,t} \end{pmatrix}, \bar{\eta}_t = \begin{pmatrix} \bar{\eta}_{M,t} \\ \bar{\eta}_{1,t} \\ \vdots \\ \bar{\eta}_{n,t} \end{pmatrix}, \bar{\eta}_{i,t} = \begin{cases} -\bar{\epsilon}_{i,t} & \text{if } \bar{\epsilon}_{i,t} < 0 \\ 0 & \text{otherwise} \end{cases} \quad \forall i. \tag{8}$$

The bars indicate firm shocks. Of course, they are related to stock return shocks through leverage,

$$\epsilon_{i,t} = (1 + LR_{i,t-1})\bar{\epsilon}_{i,t}.$$

The conditional variance covariance matrix at the firm level is

$$\bar{\Sigma}_t = E(\bar{\epsilon}_t \bar{\epsilon}_t' \mid I_{t-1}) = \begin{pmatrix} \bar{\sigma}^2_{M,t} & \bar{\sigma}_{M1,t} & \cdots & \bar{\sigma}_{Mn,t} \\ \bar{\sigma}_{M1,t} & \bar{\sigma}^2_{1,t} & \cdots & \bar{\sigma}_{1n,t} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\sigma}_{Mn,t} & \bar{\sigma}_{1n,t} & \cdots & \bar{\sigma}^2_{n,t} \end{pmatrix}, \tag{9}$$

which is modeled as

$$\bar{\Sigma}_t = \Omega\Omega' + B\bar{\Sigma}_{t-1}B' + C\bar{\epsilon}_{t-1}\bar{\epsilon}'_{t-1}C' + D\bar{\eta}_{t-1}\bar{\eta}'_{t-1}D'. \tag{10}$$

In "VEC" notation the model becomes

$$\begin{aligned} \mathrm{VEC}(\bar{\Sigma}_t) = \Omega^{\#} + B^{\#}\mathrm{VEC}(\bar{\Sigma}_{t-1}) \\ + C^{\#}\mathrm{VEC}(\bar{\epsilon}_{t-1}\bar{\epsilon}'_{t-1}) + D^{\#}\mathrm{VEC}(\bar{\eta}_{t-1}\bar{\eta}'_{t-1}), \end{aligned} \tag{11}$$

with $\Omega^{\#} = \mathrm{VEC}(\Omega\Omega')$, $B^{\#} = B \otimes B$, $C^{\#} = C \otimes C$, and $D^{\#} = D \otimes D$. $\Omega, B, C,$ and $D$ are $n+1$ by $n+1$ constant matrices, with elements $\omega_{ij}$ and $b_{ij}$, etc. The conditional variance and covariance of each excess return are related to past conditional variances and covariances, past squared residuals and cross residuals, and past squared asymmetric shocks and cross-asymmetric shocks.[6]

Apart from its technical advantages that simplify estimation [see Engle and Kroner (1995)], the BEKK model is better suited for our purposes than alternative multivariate GARCH models. The (diagonal) VECH model of Bollerslev, Engle, and Wooldridge (1988) cannot capture volatility feedback effects at the firm level. The factor ARCH model [Engle, Ng, and Rothschild (1990)] assumes that the covariance matrix is driven by the conditional variance process of one portfolio (the market portfolio), making it impossible to test for firm-specific leverage effects. The constant correlation model of Bollerslev (1990) restricts the correlation between two asset returns to be constant over time. Braun, Nelson, and Sunier (1995) use univariate asymmetric GARCH models coupled with a specification for the conditional beta that accommodates asymmetry. As we suggest above, it is more natural to model asymmetry in covariances, as is possible in the BEKK framework.

One drawback of the BEKK model is the large number of parameters that must be estimated. For a system of $m$ equations, there are $(9m^2 + m + 2)/2$ parameters. For example, a system of 4 equations has 75 parameters. To keep the size of the parameter space manageable, we impose additional constraints. We assume that lagged market-level shocks and variables enter all conditional variance and covariance equations, but that individual portfolio shocks and variables have explanatory power only for their own variances and covariances with the market.

The parameter matrices $B, C$, and $D$ now have the form, for example,

$$B = \begin{pmatrix} b_{MM} & 0 & \cdots & 0 \\ b_{M1} & b_{11} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_{Mn} & 0 & \cdots & b_{nn} \end{pmatrix}.$$

This reduces the parameter space considerably while leaving flexibility in modeling the processes of all conditional variances and covariances with the market. For a system of 4 equations, there are 39 parameters instead of 75. We analyze the implied volatility dynamics in more detail in the next subsection.

[6] Note that the asymmetric shock is defined using the negative shocks, as opposed to Glosten, Jagannathan, and Runkle (1993), who use positive shocks. This is consistent with the idea that the strong form of asymmetric volatility, discussed above, is most likely to arise from the direct leverage effect, see below. We also estimated a model where positive and negative shocks were simply allowed to have different coefficients. Since it yielded qualitatively similar results, we do not report it here.

Given this firm-level volatility model, leverage effects are now easily incorporated. Define

$$l_t = \begin{pmatrix} 1 + LR_{M,t} & 0 & \cdots & 0 \\ 0 & 1 + LR_{1,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 + LR_{n,t} \end{pmatrix}.$$

Then $\Sigma_t = E(\epsilon_t \epsilon_t' | I_{t-1}) = l_{t-1} \overline{\Sigma}_t l_{t-1}'$. Hence, if firm variances were to be constant, leverage affects conditional variances and covariances exactly as in the Christie (1982) model. If firm variances move around, their changes have a higher impact on stock return volatility when leverage is also higher. Note that the model remains econometrically attractive, guaranteeing symmetry, and hence positive definiteness, as in a standard BEKK model.

### 1.3 Empirical hypotheses

**1.3.1 Asymmetry, volatility feedback, and leverage.** If $B = C = D = 0$ (no GARCH, no volatility feedback), the model reduces to the Christie (1982) leverage model under riskless debt. That is,

$$\sigma_{i,t+1}^2 = (1 + LR_{i,t})^2 \cdot \overline{\sigma}_i^2 \quad \text{for } i = M, 1, \ldots, n. \tag{12}$$

We provide tests of this hypothesis, but also separately test for the significance of GARCH effects ($B = C = 0$) and asymmetries in the firm variance process ($D = 0$). The latter model would constitute a GARCH model where all asymmetric effects are accounted for by leverage effects. That is, a simple likelihood ratio test can determine whether volatility feedback (which must enter through the parameters in $D$) is statistically significant. Furthermore, when the asymmetric effects at the firm level are purely caused by the volatility feedback effect, we would expect the diagonal elements on $D$ (except $d_{MM}$) to be zero—we test this hypothesis as well.

To gain further insight into the relative importance of feedback effects versus leverage effects, let's analyze the volatility dynamics in more detail. Using the relation between firm and stock return shocks, we can write volatility at the stock return level as

$$\begin{aligned} \Sigma_t = {} & l_{t-1}(\Omega\Omega')l_{t-1}' + B_{t-1}^* \Sigma_{t-1}' (B_{t-1}^*)' \\ & + C_{t-1}^* \epsilon_{t-1} \epsilon_{t-1}' (C_{t-1}^*)' + D_{t-1}^* \eta_{t-1} \eta_{t-1}' (D_{t-1}^*)', \end{aligned} \tag{13}$$

where, for example,

$$B_{t-1}^* = l_{t-1} B l_{t-2}^{-1}.$$

The volatility at the market level consequently follows a univariate, leverage-adjusted, asymmetric GARCH model:

$$\sigma_{M,t}^2 = (l_{M,t-1})^2\, \omega_{MM} + \left(\frac{l_{M,t-1}}{l_{M,t-2}}\right)^2 \left(b_{MM}^2 \sigma_{M,t-1}^2 + c_{MM}^2 \epsilon_{M,t-1}^2 + d_{MM}^2 \eta_{M,t-1}^2\right),$$

where $\omega_{MM}$ is the first diagonal element of $\Omega\Omega'$ and $l_{i,t-1}$ represents the relevant diagonal element in $l_{t-1}$. Apart from the "Christie term," leverage enters in two ways in the conditional variance model. First, the historical leverage level of the market is embedded in $\sigma_{M,t-1}^2$, $\epsilon_{M,t-1}^2$, and $\eta_{M,t-1}^2$, so that similar firm shocks generate larger volatility effects whenever leverage happens to be higher. Second, an increase in leverage at time $t-1$ increases the normal GARCH effect with the ratio $(l_{M,t-1}/l_{M,t-2})^2$. Volatility at the portfolio level is equally intuitive. Given the symmetry of the model, we only consider the terms containing past variances:

$$\left(\frac{l_{i,t-1}}{l_{M,t-2}}\right)^2 b_{Mi}^2 \sigma_{M,t-1}^2 + 2\frac{l_{i,t-1}^2}{l_{i,t-2}\, l_{M,t-2}} b_{ii} b_{Mi} \sigma_{iM,t-1} + \left(\frac{l_{i,t-1}}{l_{i,t-2}}\right)^2 b_{ii}^2 \sigma_{ii,t-1}^2.$$

The first term is the only term that would be present in a factor ARCH model. Since $\sigma_{M,t-1}^2$ reflects market leverage at time $t-2$, the model adjusts the factor ARCH effect upward only when the current portfolio leverage is higher than the past market leverage level. The second term (involving the past covariance) and the third term (involving the past idiosyncratic variance) are adjusted similarly. The second term reveals the importance of interaction terms, such as $\eta_i \eta_M$ and $\epsilon_i \epsilon_M$, even in the variance equations. To present the volatility dynamics graphically, we therefore make use of news impact surfaces as in Kroner and Ng (1998).

The news impact surface graphs the conditional variance as a function of the shocks, keeping the other inputs to the conditional variance equation (conditional variances and covariances) constant at their unconditional means. In all of our graphs, we will normalize the value when the shocks are zero to be zero. We call the effect of the $\epsilon$ and $\eta$ shocks the "direct effect." Of course, our variance equation also incorporates leverage ratios. We augment the news impact curves with the effect of changes in leverage using a second-order Taylor approximation to the nonlinear relation between leverage ratios and shocks, evaluated at the sample mean.[7]

[7] Since we compute returns as the logarithm of gross returns, the level of leverage ratio as a function of the return shock is

$$LR(\epsilon_t) = \overline{LR}\left[1 + \epsilon_t^2 - \epsilon_t\right],$$

where $\overline{LR}$ is evaluated at the sample mean of leverage ratios.

**1.3.2 Covariance and beta asymmetry.** The covariance dynamics implied by the model can be written as

$$\begin{aligned}\sigma_{Mi,t} = {} & l_{i,t-1} l_{M,t-1} \omega_{iM} + b^*_{MM,t-1} b^*_{Mi,t-1} \sigma^2_{MM,t-1} \\ & + b^*_{MM,t-1} b^*_{ii,t-1} \sigma^2_{Mi,t-1} + c^*_{MM,t-1} c^*_{Mi,t-1} \epsilon^2_{M,t-1} \\ & + c^*_{MM,t-1} c^*_{ii,t-1} \epsilon_{M,t-1} \epsilon_{i,t-1} + d^*_{MM,t-1} d^*_{Mi,t-1} \eta^2_{M,t-1} \\ & + d^*_{MM,t-1} d^*_{ii,t-1} \eta_{M,t-1} \eta_{i,t-1},\end{aligned}$$

where, for example,

$$b^*_{MM,t-1} = \frac{l_{M,t-1}}{l_{M,t-2}} b_{MM}$$

$$b^*_{Mi,t-1} = \frac{l_{i,t-1}}{l_{M,t-2}} b_{Mi}.$$

These dynamics are quite general. There is a constant term that reflects leverage effects as in Christie (1982). The first variance term represents a "factor ARCH" term. When the conditional market variance was high last period, so will be the current market variance and all covariances between stock returns and the market return. The leverage adjustments correct for the fact that leverage may have changed since last period. Hence there is an indirect source of a leverage effect in the covariance equation: with a positive market shock, market leverage decreases and the "factor ARCH" effect is downweighted and vice versa. Furthermore, since the ratio $l_{i,t-1}/l_{M,t-2}$ multiplies the market variance term, high leverage firms will tend to exhibit larger "factor ARCH" effects. The second term is a persistence term; shocks to the covariance persist over time and they are scaled up or down by changes in both market and firm leverage. Finally, the shock terms allow for different effects on the covariance depending on the particular combination of market and individual shocks. Generally we would like our estimate of the conditional covariance to be increased when these shocks are of the same sign and to be decreased otherwise. Ideally the model should accommodate a different covariance response depending on whether the underlying shocks are positive or negative.

To see how this generalized BEKK model accomplishes this, let $u_i = |\epsilon_i|$ and consider the covariance response to all possible combinations of positive and negative market and individual shocks. We ignore the leverage corrections in the table.

| | $\epsilon_M > 0$ | $\epsilon_M < 0$ |
|---|---|---|
| $\epsilon_i > 0$ | $c_{MM}c_{Mi}u_M^2 + c_{MM}c_{ii}u_M u_i$ | $c_{MM}c_{Mi}u_M^2 - c_{MM}c_{ii}u_M u_i$ $+d_{MM}d_{Mi}u_M^2$ |
| $\epsilon_i < 0$ | $c_{MM}c_{Mi}u_M^2 - c_{MM}c_{ii}u_M u_i$ | $c_{MM}c_{Mi}u_M^2 + c_{MM}c_{ii}u_M u_i$ $+d_{MM}d_{Mi}u_M^2 + d_{MM}d_{ii}u_M u_i$ |

Since the sign of the different parameters is not restricted, virtually any pattern is possible. Suppose that all parameters are positive. The BEKK model then revises downward the covariance when shocks have opposite signs and the off-diagonal responses are uniformly smaller than the diagonal responses in the same column. Even so, the response need not be negative, because of the "factor ARCH" term. If the coefficients are positive (they need not be), the covariance response is more pronounced when the market shock is negative, which is how volatility feedback is accommodated.

The generality of these dynamics comes at a cost, in that the BEKK model imposes nonlinear restrictions on the parameters, which translates into restrictions on the particular magnitude of the responses (see table). Furthermore, the restrictions imply that covariances and variances are partially driven by the same parameters. Nevertheless, it is possible for the model to generate volatility asymmetry in response to market shocks without generating covariance asymmetry or to simultaneously generate reverse covariance asymmetry. Consistent with the volatility feedback model, the strength of covariance and volatility asymmetry is positively correlated when the parameter $d_{Mi}$ is positive.

As noted above, volatility feedback at the firm level is likely to be accompanied by covariances that increase more when the shock is negative than when it is positive. Whereas this is directly testable using the parameter estimates, we also produce news impact surfaces for the covariances. We investigate whether covariance asymmetry translates into strong volatility feedback effects and whether it is more pronounced for firms with high systematic risk.

Given that most recent research has focused on asymmetries in betas, we examine whether the conditional betas implied by our model exhibit leverage effects. To do so, we create approximate news impact surfaces for the $\beta$'s. This can be accomplished by combining the impact of shocks on conditional variances and covariances.

## 1.4 Specification tests

**1.4.1 Generic tests.** We conduct tests of the specification of the conditional means, variances, and covariances. These tests are indicated by MEAN, VAR, and COV, respectively. All tests use the standardized residuals: $z_t$, which are computed as $(P_t')^{-1}\epsilon_t$, with $\Sigma_t = P_t'P_t$. That is, $z_t$

is an $N(0, I)$ vector conditional on time $t-1$ information and the model being well specified. For each test and most other tests below we use the generalized method of moments [Hansen (1982)] to test moment implications of a well-specified model, which are of the general form

$$E[V_t | I_{t-1}] = 0,$$

with $V_t$ a vector stochastic process. The resulting test statistic has an asymptotic chi-square distribution with degrees of freedom equal to the dimension of $V_t$. The use of estimated residuals and the size of our sample may imply that the actual small sample distribution of the test statistics is no longer a chi-square distribution. Monte Carlo results in Bekaert and Harvey (1997) suggest that the small sample distribution of the tests may have more mass in the right tail so that we overreject at the asymptotic critical values.

The conditional mean test, MEAN, sets

$$V_t = \begin{bmatrix} z_{it} \\ z_{it} \cdot z_{it-j} \end{bmatrix} \quad j = 1,2,3; \quad i = M,1,2,3.$$

MEAN tests the serial correlation properties of the standardized residuals and is done for each portfolio separately and jointly for all portfolios.

For the conditional variance tests, VAR, we introduce the variable $q_{i,t} = z_{i,t}^2 - 1$ and we let

$$V_t = \begin{bmatrix} q_{it} \\ q_{it} \cdot q_{it-j} \end{bmatrix} \quad j = 1,2,3; \quad i = M,1,2,3.$$

Again the test is done separately for the different portfolios and jointly for all portfolios. Finally, to test the conditional covariance specification, consider the variable

$$W_{i,t} = \frac{\epsilon_{i,t}\epsilon_{M,t}}{\sigma_{iM,t}} - 1 \quad i = 1,2,3.$$

We let

$$V_t = \begin{bmatrix} W_{it} \\ W_{it} \cdot W_{it-j} \end{bmatrix} \quad j = 1,2,3;$$

for each portfolio $i$ and all portfolios jointly.

**1.4.2 Testing the CAPM assumption.** The MEAN test partially tests the CAPM assumption. If other risks are priced, the mean of the residual may not be zero. However, this test may not be powerful enough to detect particular deviations from the CAPM and we provide a number of alternative tests. Our first CAPM test, $CAPM_1$, provides a simple test of whether leverage plays a role in the conditional mean. Bhandari (1988) shows that leverage is cross-sectionally priced in U.S. stock returns. Moreover, if debt is not riskless, leverage ratios may enter the conditional mean.

We put

$$V_t = \begin{bmatrix} z_{it} \cdot LR_{M,t-1} \\ z_{it} \cdot LR_{i,t-1} \\ z_{Mt} \cdot LR_{M,t-1} \end{bmatrix} \quad i = 1,2,3. \tag{14}$$

for a total of 7 restrictions.

Second, since we use weekly data (see below), there may be serial correlation in the portfolio returns, for example because of liquidity problems, that is not captured by the CAPM model. The MEAN test implicitly tests the serial correlation properties of the returns, but we also provide a more explicit test by putting

$$V_t = \begin{bmatrix} z_{it} \cdot r_{M,t-1} \\ z_{it} \cdot r_{i,t-1} \\ z_{Mt} \cdot r_{M,t-1} \end{bmatrix} \quad i = 1,2,3. \tag{15}$$

The $CAPM_2$ test has seven restrictions and also tests whether past market portfolio returns predict future portfolio residuals, which may be the case if liquidity problems prevent information from being incorporated quickly into the prices of smaller stocks.

Third, previous research [see, e.g., Harvey (1991), Bekaert and Harvey (1995), and DeSantis and Gerard (1997)] has uncovered time variation in the prices of risk for a large number of equity markets across the world. It is likely that the price of risk varies with the business cycle [see Campbell and Cochrane (1995)]. In a previous version of this article we also considered a more general model with a time-varying price of risk. The model yielded similar qualitative results.

**1.4.3 Interest rate effects.** As stressed by Christie (1982), and confirmed by a number of empirical studies, interest rates are good predictors of stock market volatility. Interest rate changes also affect the market value of debt and hence leverage ratios. Since we use book values of debt in the empirical work below, the measurement error in leverage ratios may be correlated with interest rates.

We examine remaining interest rate effects in both conditional variances and covariances. $INT_1$ sets

$$V_t = \left[ q_{i,t} \cdot r^f_{t-1,t} \right] \quad i = M,1,2,3, \tag{16}$$

and $\text{INT}_2$ lets

$$V_t = \left[W_{i,t} \cdot r^f_{t-1,t}\right] \quad i = 1,2,3, \tag{17}$$

Finally, if debt is not riskless, the conditional mean for equity returns depends on the risk-free rate through the expected excess return on debt. Hence, for $\text{INT}_3$, we set

$$V_t = \left[z_{i,t} \cdot r^f_{t-1,t}\right] \quad i = M,1,2,3. \tag{18}$$

## 2. Empirical Results

### 2.1 The Nikkei 225 data

Our data consists of daily observations on the (dividend-adjusted) prices and market capitalization of the firms in the Nikkei 225 index. In addition, we have biannual data on their book value of debt. The sample period is from January 1, 1985 to June 20, 1994. Stocks that are not in the Nikkei 225 index over the whole period or do not have debt data are discarded. There are 172 stocks left in the sample. We construct three portfolios of five stocks each, representing a low leverage, medium leverage, and high leverage portfolio. To do so, daily leverage ratios are calculated, with the missing debt dataset equal to the last available data point. Then we rank all firms according to their average leverage ratios. The leverage portfolios consist of five stocks with the lowest, the medium, or the highest leverage ratios, respectively, excluding commercial banks.[8] The portfolio leverage ratios are then calculated as the total debt over the total capitalization of the portfolio. For the market leverage ratio, we use the ratio of total debt over total capitalization of the 172 stocks in the sample. Finally, we extract weekly observations on leverage and stock returns from the daily data.

The leverage ratio data are measured with error because the debt value is a book instead of a market value and because it is only updated every 6 months. Moreover, the substantial time variation observed in the capital structure of a firm over a 10-year period may make a classification based on leverage difficult. Nevertheless, Table 2 shows that our portfolios have very distinct leverage ratios over the full sample period. In particular, the leverage ranking is preserved not only on average but at every point in time. Their return characteristics do not appear significantly different.

For the short-term interest rate, we use the 1-month Gensaki rate, which is the yield on bond repurchase contracts.[9] As noted in Dickson,

[8] Portfolios are constructed analogous to the construction of the Nikkei 225 index, that is, the total value of a portfolio is the sum of the value of individual stocks with dividends reinvested.

[9] We divide the annualized rate by 5200 to express it as a weekly yield. Implicitly we assume a flat term interest rate structure at the very short end of the maturity spectrum.

**Table 2**
**Summary information on the leverage portfolios**

| | High leverage | Medium leverage | Low leverage |
|---|---|---|---|
| Capitalization (¥, $10^{11}$) | 46.23 | 9.19 | 28.74 |
| Unconditional beta | 1.1122 | 1.1088 | 0.6244 |

| | Market | High leverage | Medium leverage | Low leverage |
|---|---|---|---|---|
| Returns of portfolios | | | | |
| Maximum | 10.269 | 13.947 | 12.358 | 22.213 |
| Minimum | − 12.790 | − 13.956 | − 21.849 | − 15.378 |
| Mean | 0.126 | 0.165 | 0.108 | 0.209 |
| Std. deviation | 2.671 | 3.843 | 3.831 | 3.426 |
| AC(1) | 0.028 | − 0.023 | 0.015 | − 0.061 |
| AC(2) | 0.050 | 0.030 | 0.023 | 0.015 |
| AC(3) | 0.090 | 0.051 | 0.136 | 0.006 |
| AC(4) | − 0.021 | 0.003 | − 0.030 | − 0.070 |
| AC(5) | 0.008 | 0.053 | − 0.033 | 0.012 |
| Leverage ratios of portfolios | | | | |
| Maximum | 2.430 | 10.844 | 1.402 | 0.362 |
| Minimum | 0.753 | 2.971 | 0.393 | 0.137 |
| Mean | 1.354 | 5.796 | 0.835 | 0.210 |
| Std. deviation | 0.445 | 1.895 | 0.263 | 0.054 |

The leverage portfolios consist of five stocks each with respectively the highest, the medium, and the lowest average leverage ratios over the sample period. Commercial banks are excluded. The sample period is January 1985 to June 1994 and the data are sampled weekly. The capitalization row reports the average market capitalization over the sample period. Returns are computed as the logarithm of dividend-inclusive gross returns and their characteristics are reported in percent. AC($i$) stands for autocorrelation of order $i$.

Fuchida, and Nishizawa (1990), the Gensaki market was the first market for short-term investment in Japan with rates determined freely by supply and demand of funds.

## 2.2 Estimation and specification tests

To estimate the model in Equations (5)–(11), we assume that the innovations are conditionally normal. We obtain quasi-maximum likelihood estimates of the parameters with White (1980) standard errors [see also Bollerslev and Wooldridge (1992)]. The price of risk is estimated to be 0.058, with a standard error of 0.0844. To make sure that the standard errors are not affected by possible numerical problems, we also computed them with the Hessian matrix approximated by the cross product of the gradients. There do not appear to be any large changes in the standard errors for the coefficients of the variance equation. However, the standard error of the price of risk is a much bigger 3.874. We suspect that our model has underestimated the price of risk [see Merton (1980) for a discussion on the difficulty of estimating expected returns from a finite sample of data]. To obtain an alternative benchmark, consider the unconditional price of risk that would result if

**Table 3**
**Tests of the model specification**

| Test statistics | MEAN | VAR | COV | CAPM$_1$ | CAPM$_2$ | INT$_1$ | INT$_2$ | INT$_3$ |
|---|---|---|---|---|---|---|---|---|
| General | 15.029 | 15.137 | 11.641 | 16.152 | 4.339 | 1.379 | 0.406 | 0.538 |
| | (0.522) | (0.515) | (0.475) | (0.024) | (0.740) | (0.848) | (0.939) | (0.970) |
| Market portfolio | 8.670 | 5.307 | | | | | | |
| | (0.070) | (0.257) | | | | | | |
| High leverage | 2.417 | 6.069 | 4.477 | | | | | |
| | (0.659) | (0.194) | (0.345) | | | | | |
| Medium leverage | 1.327 | 0.498 | 0.960 | | | | | |
| | (0.857) | (0.974) | (0.916) | | | | | |
| Low leverage | 1.147 | 2.160 | 6.873 | | | | | |
| | (0.887) | (0.706) | (0.143) | | | | | |

The first row is the value of the statistic and the second row contains the $p$-value (in parentheses). MEAN tests whether the means and three autocorrelations of the standardized residuals are zero; VAR tests whether the means of the squared standardized residuals are 1 and three autocorrelations of the squared standardized residuals are zero. The MEAN and VAR tests are asymptotically distributed $\chi^2$ (4) for individual portfolios and $\chi^2$ (16) for the general test. COV provides an analogous test for the cross residuals scaled by the conditional covariance and is asymptotically distributed as a $\chi^2$ (3) for individual portfolio tests and $\chi^2$ (12) for the general test. The two CAPM statistics test the orthogonality of scaled residuals to respectively past leverage ratios and past returns (CAPM$_1$ and CAPM$_2$ are $\chi^2$ (7)). Finally, the INT tests focus on the orthogonality of respectively standardized variances, covariances, and residuals to past interest rates, they all have asymptotic $\chi^2$ (4) distributions.

variances were not time varying, that is

$$Y^U = \frac{E\left[\left(r_{M,t} - r^f_{t-1,t}\right)\left(1 + LR_{M,t-1}\right)\right]}{\sigma_M^2}. \tag{19}$$

Estimating both the numerator and denominator by their sample counterparts, we find $Y^U = 1.896$. This number will help us to illustrate the economic implications of the model in the following section.

Before we discuss the estimation results, we want to ensure that the model is well specified. The specification tests discussed in Section 1.4 are reported in Table 3. The MEAN, VAR, and COV statistics for portfolios individually and jointly reveal no evidence against the model. No rejections occur at the 5% level. There do not appear to be interest rate effects in the variance, covariance, and mean that we fail to capture. However, there is some weak evidence against the CAPM model. Leverage ratios may have some remaining predictive power, but the test does not reject at the 1% level (CAPM$_1$).

### 2.3 Volatility persistence, volatility feedback, and leverage effects

**2.3.1 Likelihood ratio tests.** The simultaneous presence of leverage ratios, asymmetric shocks, and volatility persistence makes our model more general than previous volatility specifications. In Table 4, we present a number of likelihood ratio tests to determine the potential validity of the more restrictive models discussed in Section 1.3.1.

**Table 4**
**Likelihood tests for various models**

| Null hypothesis | Test statistic | Degrees of freedom | $P$ Value |
|---|---|---|---|
| $D = 0$ | 85.78 | 7 | 8.89e-16 |
| $d_{22} = d_{33} = d_{44} = 0$ | 60.93 | 3 | 3.73e-13 |
| $B = C = 0$ | 244.23 | 14 | 0 |
| $B = C = D = 0$ | 384.54 | 21 | 0 |

This table presents the likelihood ratio tests of various restrictions on the model. $B, C$, and $D$ are parameter matrices in the BEKK covariance matrix process for the lagged (co) variances, return shocks, and asymmetric shocks, respectively. The first test examines the significance of asymmetric effects at the firm level. The second line reports a test on the presence of firm level asymmetries not caused by market shocks. The third line represents a test of the GARCH(1,1) volatility structure and the fourth line can be viewed as a test of the Christie (1982) leverage model.

First, the $B = C = D = 0$ restrictions basically reduce our model to that of the Christie (1982) leverage effect volatility model in Equation (12). The multivariate GARCH structure is often ignored in the literature focusing on individual firms and leverage effects. Clearly, leverage variables alone cannot account for the volatility behavior of the Japanese stock returns. We also reject the GARCH effects restriction ($B = C = 0$).

Second, since both the asymmetric shocks ($\eta$) and the leverage ratios give rise to asymmetric volatility, it may be superfluous to have both. The overwhelming rejection of $D = 0$ shows that asymmetric volatility still exists even after leverage effects have been "filtered out." In part, the presence of leverage ratios may simply enable our model to capture strong asymmetry. We investigate whether the model generates strong asymmetry below.

Finally, we reject the hypothesis that the diagonal elements on $D$ (except $d_{MM}$) are zero. Since the proper measure of risk for individual portfolios is the conditional covariance in our CAPM framework [see Equation (5)], all we need to generate volatility feedback is a dependence of the covariance and firm volatility on market shocks. The rejection of this hypothesis may indicate that the asymmetric effects are richer than just the feedback effect. Alternatively, since our firm shocks do not reflect purely firm-specific shocks, it may simply reflect correlation between market and firm shocks.

**2.3.2 Volatility dynamics.** For completeness, Table 5 reports the original parameter estimates. Table 6 shows the more informative VEC-form coefficients of the variance equations with standard errors in parentheses. From Equations (9) and (11), these coefficients show the impact on variance at the firm level. To see the impact on the variance at the equity level, we need to incorporate the leverage effect as shown in Equation (13), since $\Sigma_t = E(\epsilon_t \epsilon_t' | I_{t-1}) = l_{t-1} \tilde{\Sigma}_t l_{t-1}'$.

**Table 5**
**Estimated variance equation parameter matrix**

| | Parameters | | | | Standard errors | | | |
|---|---|---|---|---|---|---|---|---|
| Ω | 0.00271 | 0 | 0 | 0 | 0.0008 | 0 | 0 | 0 |
| | 0.00093 | −0.0005 | 0 | 0 | 0.0003 | 0.00014 | 0 | 0 |
| | 0.00312 | 0.00021 | −0.0024 | 0 | 0.0014 | 0.00053 | 0.00067 | 0 |
| | 0.00495 | 0.00282 | −1E-05 | −0.0047 | 0.0015 | 0.00113 | 0.00088 | 0.0017 |
| B | 0.89429 | 0 | 0 | 0 | 0.0262 | 0 | 0 | 0 |
| | −0.0195 | 0.96144 | 0 | 0 | 0.0086 | 0.00984 | 0 | 0 |
| | −0.0408 | 0 | 0.92237 | 0 | 0.0595 | 0 | 0.01414 | 0 |
| | −0.0939 | 0 | 0 | 0.94717 | 0.0583 | 0 | 0 | 0.02246 |
| C | 0.2846 | 0 | 0 | 0 | 0.02726 | 0 | 0 | 0 |
| | 0.02576 | 0.21853 | 0 | 0 | 0.01595 | 0.0368 | 0 | 0 |
| | −0.1348 | 0 | 0.32517 | 0 | 0.06079 | 0 | 0.03047 | 0 |
| | 0.46463 | 0 | 0 | 0.02959 | 0.06783 | 0 | 0 | 0.042911 |
| D | 0.38643 | 0 | 0 | 0 | 0.04435 | 0 | 0 | 0 |
| | 0.08207 | 0.13279 | 0 | 0 | 0.02324 | 0.06396 | 0 | 0 |
| | 0.79189 | 0 | −0.0504 | 0 | 0.06961 | 0 | 0.07211 | 0 |
| | −0.2023 | 0 | 0 | 0.30117 | 0.06838 | 0 | 0 | 0.04341 |

This table lists the estimated variance-covariance equation parameter matrices of the BEKK model as defined in Equation (10). Heteroscedasticity-consistent standard errors of corresponding elements are listed in the right column.

**Table 6**
**Impact of variables on conditional variances**

| Equation | $\sigma^2_{M,t-1}$ | $\sigma^2_{i,t-1}$ | $\varepsilon^2_{M,t-1}$ | $\varepsilon^2_{i,t-1}$ | $\eta^2_{M,t-1}$ | $\eta^2_{i,t-1}$ |
|---|---|---|---|---|---|---|
| Market | 0.7998 | | 0.0810 | | 0.1493 | |
| | (0.0468) | | (0.0155) | | (0.0343) | |
| High | 0.0004 | 0.9244 | 0.0007 | 0.0478 | 0.0067 | 0.0176 |
| | (0.0003) | (0.0189) | (0.0008) | (0.0160) | (0.0038) | (0.0169) |
| Medium | 0.0017 | 0.8508 | 0.0182 | 0.1057 | 0.6271 | 0.0025 |
| | (0.0049) | (0.0261) | (0.0164) | (0.0198) | (0.1103) | (0.0073) |
| Low | 0.0088 | 0.8971 | 0.2159 | 0.0009 | 0.0409 | 0.0907 |
| | (0.0109) | (0.0425) | (0.0630) | (0.0025) | (0.0277) | (0.0261) |

This table presents the impact on conditional *variances* given a change in the variables listed in columns, while holding other variables constant; that is, the coefficients are computed using the VEC representation of the BEKK model estimates and the delta method. We do not report the coefficients on interaction terms such as $\varepsilon_{i,t-1}\varepsilon_{M,t-1}$. Heteroscedasticity-consistent standard errors are in parentheses and are computed using the variance-covariance matrix of the parameter estimates.

The parameter estimates in Table 6 imply that conditional volatility is quite persistent both at the market level and at the portfolio level. The coefficient on lagged volatility is always significant and between 0.7998 and 0.9244. At the market level, the asymmetry of the volatility response to return shocks is significant and pronounced. The coefficient on the squared return shock is 0.0810, while the coefficient on the squared asymmetric shock term is 0.1493. Both are statistically significant. As a result, a positive unit shock raises conditional variance by 0.0810, but a negative shock raises conditional variance by 0.2303. In fact, one disadvantage of the BEKK structure is that it does not accommodate the strong form of asymmetric volatility, where positive return shocks decrease conditional volatility [see Equation (1) and the associated footnote]. Even so, the model yields asymmetry at the firm level when leverage effects have been filtered out. The asymmetry is primarily caused by portfolio shocks for the low leverage portfolio and by market shocks for the high leverage portfolio. The medium leverage portfolio exhibits weak asymmetry for both shocks. Generally the "factor ARCH" effects are weak when one focuses on the variance terms, but stronger for the shock terms.

The parameter coefficients ignore the interaction effects between different shocks and the leverage effect. A complete picture of the response of volatility to shocks is contained in the news impact curve in Figure 2 for the market portfolio and the news impact surfaces in Figure 3 for the various portfolios. In Figure 2, the change in market variance is plotted against return shocks from $-10\%$ to $10\%$. The curve without leverage holds leverage ratios constant at the sample mean. It is obvious that the leverage effect magnifies the asymmetry, but its influ-

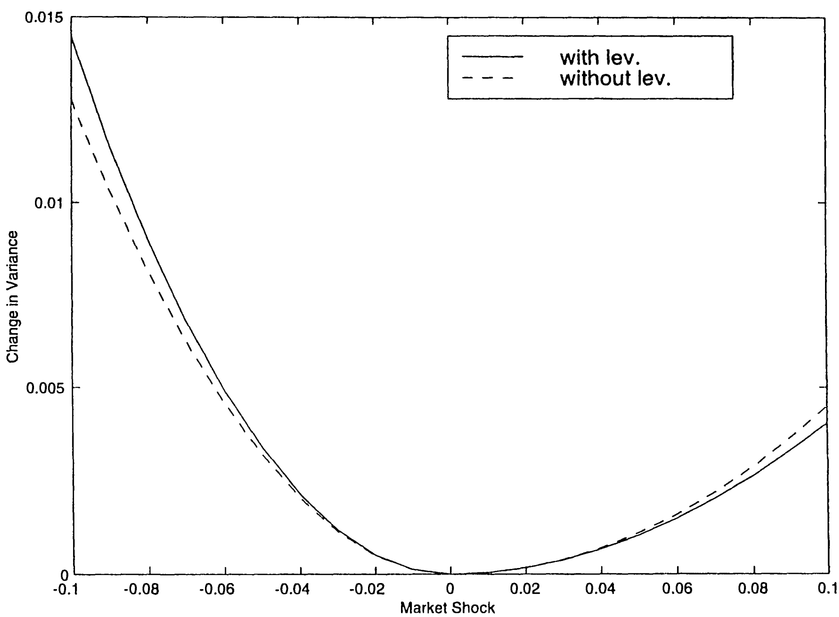


**Figure 2**
**Variance impact curve for the market portfolio**
This figure shows the market shock impact on market variance with or without incorporating the change in leverage level. The shocks are at the firm, not equity, level. Leverage ratios are evaluated at the sample mean except when leverage effects are taken into account (solid line) and $LR(\epsilon_t) = \overline{LR}[1 + \epsilon^2 - \epsilon]$, where $\overline{LR}$ is the sample mean.

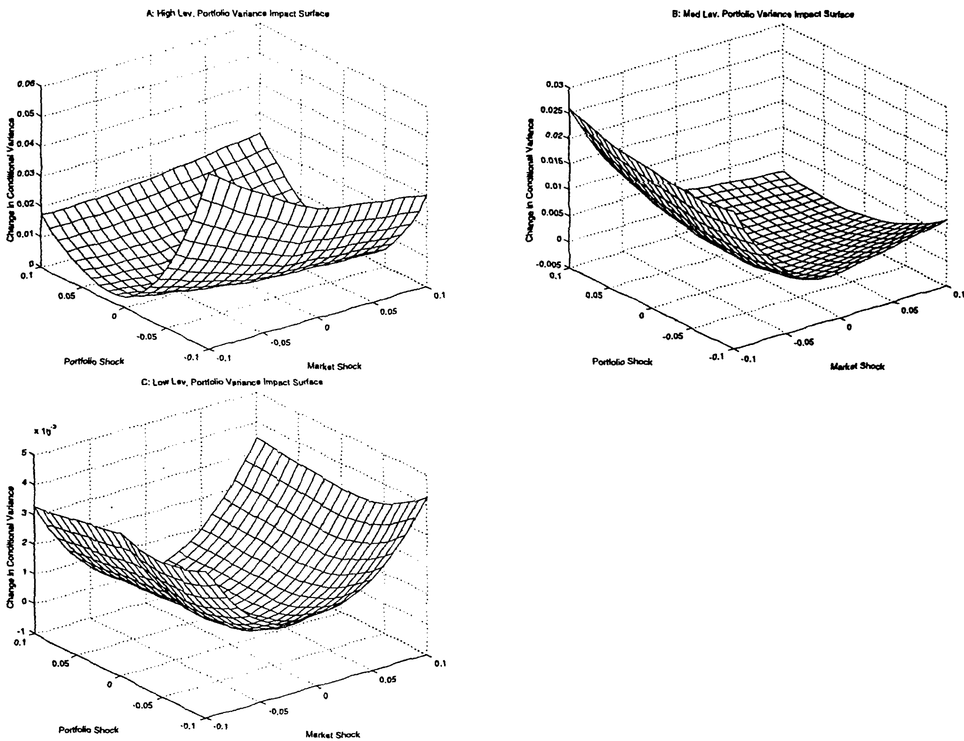


**Figure 3**
**Variance impact surfaces for leverage portfolios**
This figure shows the variance impact surfaces for the three leverage portfolios. Both portfolio and market shocks range from $-10\%$ to $+10\%$. Past leverage ratios are evaluated at the sample mean $\overline{LR}$ and current leverage ratios are approximated by $\overline{LR}[1 + \epsilon^2 - \epsilon]$.

ence is secondary when compared to the feedback effect. Figure 3A clearly shows the high leverage portfolio to exhibit distinct asymmetry with respect to both market shocks and portfolio shocks. Conditional variances increase the most when both the market shocks and portfolio shocks are negative. Given the parameter estimates in Table 6, it is not surprising that the medium leverage portfolio displays strong volatility asymmetry with respect to market shocks, but very weak asymmetry with respect to portfolio shocks. The low leverage portfolio shows little asymmetry (see Figure 3C) and market shocks seem to have a bigger impact on its variance.

To conclude, our results indicate that the high and medium leverage portfolios exhibit pronounced asymmetry caused by market shocks. For the low leverage portfolio, the asymmetry seems economically less significant. For the volatility feedback story to explain the asymmetry in the high and medium leverage portfolios, negative shocks at the market level must lead to an increase of conditional covariances between the market and these portfolios. We examine this issue in the next section.

### 2.4 Conditional covariances

At the portfolio level, the conditional covariance plays an important role in determining the expected excess return and volatility feedback according to the time-varying risk premium theory. How do return shocks and leverage ratios affect the conditional covariance? Table 7 summarizes the VEC-form of the estimated coefficients and their standard errors (in parentheses). Again, the VEC-form coefficients show directly the impact of different variables on covariance at the firm level. From Table 7 we see that the conditional covariances between the market portfolio and the leverage portfolios are very persistent. The coefficients on the lagged covariance are 0.8598, 0.8249, and 0.8470, respectively, for the high, medium, and low leverage portfolios.

Of interest, the factor ARCH effect seems rather weak. Current

**Table 7**
**Impact of variables on conditional covariances**

| Equation | $\sigma^2_{m,t-1}$ | $\sigma_{Mi,t-1}$ | $\varepsilon^2_{M,t-1}$ | $\varepsilon_{i,t-1}\varepsilon_{M,t-1}$ | $\eta^2_{M,t-1}$ | $\eta_{i,t-1}\eta_{M,t-1}$ |
|---|---|---|---|---|---|---|
| High | −0.0175 | 0.8598 | 0.0073 | 0.0622 | 0.0317 | 0.0513 |
| | (0.0073) | (0.0286) | (0.0049) | (0.0125) | (0.0112) | (0.0267) |
| Medium | −0.0365 | 0.8249 | −0.0384 | 0.0925 | 0.0306 | −0.0194 |
| | (0.0523) | (0.0259) | (0.0166) | (0.0141) | (0.0366) | (0.0261) |
| Low | −0.0839 | 0.8470 | 0.1322 | 0.0084 | −0.0782 | 0.1164 |
| | (0.0501) | (0.0290) | (0.0235) | (0.0125) | (0.0251) | (0.0244) |

This table presents the impact on conditional *covariances* given a change in the variables listed in columns, while holding other variables constant; that is, the coefficients are computed using the VEC representation of the BEKK model estimates and the delta method. Heteroscedasticity-consistent standard errors are in parentheses and are computed using the variance-covariance matrix of the parameter estimates.

portfolio covariance with the market depends on past market volatility, on which the coefficients are negative, and the market shocks, where positive signs dominate. Comparing the coefficients for the $\epsilon_M \epsilon_i$ and $\eta_M \eta_i$ shocks, there is pronounced covariance asymmetry for the high and low leverage portfolios, but not for the medium leverage portfolio. Note that this only implies that covariances are much larger when both market and portfolio shocks are negative. To obtain a more complete picture of the covariance response, including the leverage effect, consider the impact surfaces in Figure 4.

The high leverage portfolio (Figure 4A) shows pronounced covariance asymmetry. Covariances only increase when the shocks are of the same sign and they increase substantially more when both are negative. Whereas covariances rarely decrease much when the shocks are of opposite sign, they decrease more if market shocks are positive. The graph for the medium leverage portfolio is similar, with the exception that covariances increase in response to a positive portfolio but negative market shock.

These patterns confirm that the volatility asymmetry documented in Section 2.3 for the high and medium leverage portfolios is closely related to the asymmetric response of the covariance with respect to market shocks. The magnitude of the effects is enhanced by the presence of the leverage variables—at least for negative portfolio shocks. The covariance responses of the low leverage portfolio do not show the asymmetric pattern we expect. Covariances increase sharply when both portfolio and market shocks are negative, but they are particularly large for large positive market shocks. One interpretation problem inherent in the impact surfaces is that part of the graph may represent shock combinations that have very low probability of actually occurring in the data. Although we choose the shocks over an empirically relevant range, well within the range of the actual returns, it is, for example, very unlikely to observe a 10% market shock and $-10\%$ portfolio shock. We will address this problem in Section 3.

### 2.5 Conditional betas

Although some authors have found a "leverage effect" in conditional betas, Braun, Nelson, and Sunier (1995) find no evidence that betas increase (decrease) in response to bad (good) news at the industry level. As Equation (4) indicates, this is a priori not so surprising. If market leverage changes simultaneously and shocks are not purely idiosyncratic, the change in market leverage may mitigate the leverage effect of the portfolio shock. The relation between market shocks and the market beta is not very transparent in this model, since shocks affect both the conditional variance and covariance in a similar way. Neverthe-

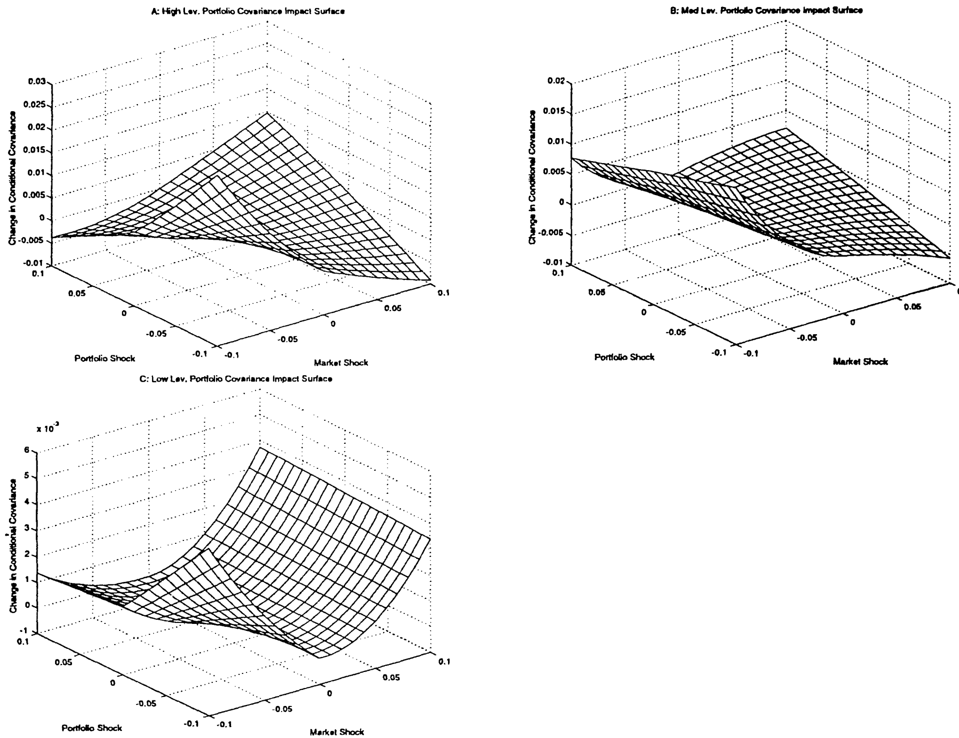


**Figure 4**
**Covariance impact surfaces for leverage portfolios**
This figure shows the covariance impact surfaces for the three leverage portfolios. Both portfolio and market shocks range from $-10\%$ to $+10\%$. Past leverage ratios are evaluated at the sample mean $\overline{LR}$ and current leverage ratios are approximated by $\overline{LR}[1 + \epsilon^2 - \epsilon]$.

less, if the leverage effect is important, we unambiguously ought to see increases in betas when market shocks are positive and portfolio shocks are negative. Figure 5A–C graphs news impact surfaces for the betas of the three leverage portfolios. The graphs for the high and medium leverage portfolios show the opposite effect. Betas sharply drop in response to highly positive market and negative portfolio shocks. This can only be true if the movement in firm betas dominates the leverage effects. Of course, the firm beta will drop in response to a positive market and negative portfolio shock, since the covariance goes down, while the market variance goes up. These numbers have to be interpreted with caution, however. The simultaneous occurrence of a large positive market shock and negative portfolio shock is very unlikely.

Overall the high and medium leverage portfolios display similar patterns for the beta response to shocks. Shocks of the same sign at the market and portfolio levels, not surprisingly, increase beta, since a positive comovement between the portfolio and the market should make the portfolio riskier. The low leverage portfolio, however, shows a different pattern. Figure 5C tells us that for positive market shocks, beta is fairly insensitive to the portfolio shocks and increases with the size of the market shocks. For negative market shocks, betas move lower, but we observe beta asymmetry of the form predicted by a standard leverage story. Betas decrease in response to positive portfolio shocks much more than in response to negative portfolio shocks. Ironically, the only portfolio for which we find something resembling a leverage effect in beta is the low leverage portfolio. Of course, the main reason for this is not the leverage effect per se, but the lack of a strong volatility feedback mechanism for this portfolio (see Sections 2.3 and 2.4). Overall, it is fair to say that the feedback effect is the dominant force driving the beta dynamics.

## 3. The Economic Significance of Asymmetric Volatility

In this section we investigate whether the volatility and covariance asymmetry phenomena implied by our parameter estimates are economically significant. To do so, we first define a "typical" firm shock for our portfolios as the mean of the absolute firm residuals implied by the parameter estimates.[10] Table 8 traces the effects of such shocks occurring for volatility, covariance, beta, and the risk premium. Of course, we

[10] Our analysis is similar to the impulse response analysis often conducted in vector autoregressions. A one standard deviation shock is the standard choice of units. Our typical shock is slightly smaller, since under normality $E|\epsilon| = \sqrt{2/\pi}\,\sigma_\epsilon$. We thank the referee for pointing this out to us.

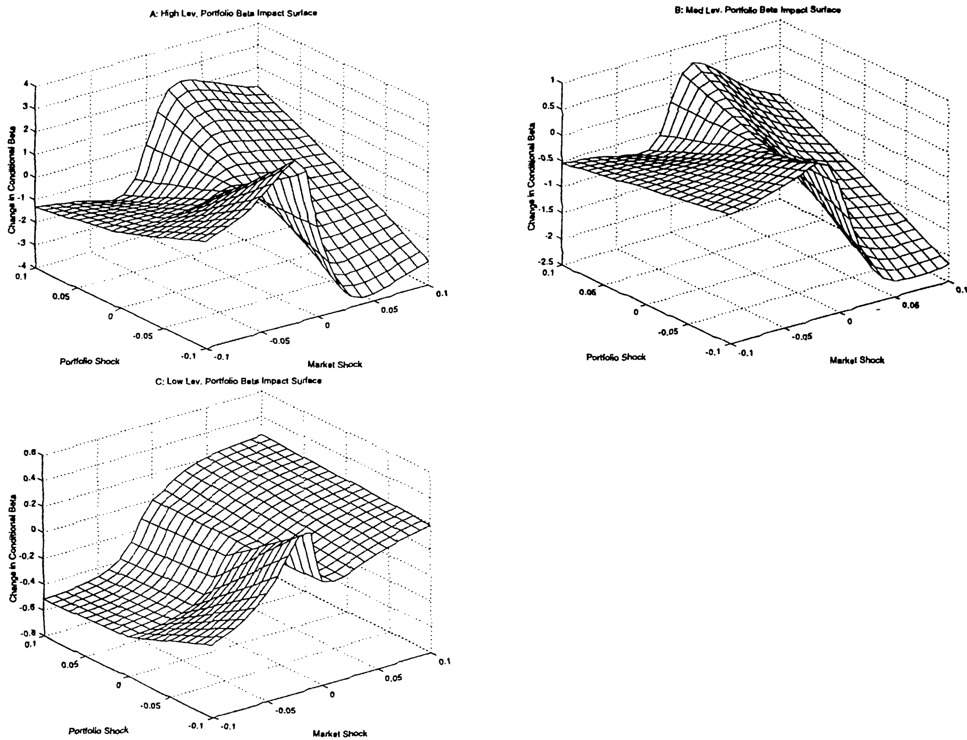


**Figure 5**
**Beta impact surfaces for leverage portfolios**
This figure shows the beta impact surfaces for the three leverage portfolios. Both portfolio and market shocks range from $-10\%$ to $+10\%$. Past leverage ratios are evaluated at the sample mean $\overline{LR}$ and current leverage ratios are approximated by $\overline{LR}[1 + \epsilon^2 - \epsilon]$.

**Table 8**
**The economic effects of typical market and portfolio shocks**

| | Firm shock | | Market shock | | Joint shock | |
|---|---|---|---|---|---|---|
| | + | − | + | − | + | − |
| Market portfolio | | | | | | |
| Volatility | | | 0.3753 | 1.3830 | 0.3753 | 1.3830 |
| Covariance | | | | | | |
| Beta | | | | | | |
| Risk premium | | | 0.1219 | 0.3980 | 0.1219 | 0.3980 |
| Risk premium (%) | | | 4.7612 | 15.5502 | 4.7612 | 15.5502 |
| High leverage portfolio | | | | | | |
| Volatility | 0.3624 | 0.7472 | 0.0217 | 0.2409 | 0.5809 | 1.5552 |
| Covariance | −1.4154 | 1.4286 | 2.6267 | 25.1805 | 21.4461 | 64.3184 |
| Beta | −0.0045 | 0.0045 | −0.0380 | −0.0880 | 0.0188 | 0.0180 |
| Risk premium | −0.0114 | 0.0115 | 0.0352 | 0.1877 | 0.1876 | 0.5014 |
| Risk premium (%) | −0.3956 | 0.3993 | 1.2232 | 6.5147 | 6.5088 | 17.4004 |
| Medium leverage portfolio | | | | | | |
| Volatility | 0.6624 | 1.0520 | 0.0447 | 1.5407 | 0.3266 | 1.8567 |
| Covariance | −2.4019 | 2.4770 | −7.8383 | 45.0595 | 16.5927 | 69.5065 |
| Beta | −0.0076 | 0.0078 | −0.0684 | −0.0301 | 0.0053 | 0.0362 |
| Risk premium | −0.0193 | 0.0200 | −0.0498 | 0.3474 | 0.1479 | 0.5434 |
| Risk premium (%) | −0.6894 | 0.7110 | −1.7754 | 12.3816 | 5.2713 | 19.3646 |
| Low leverage portfolio | | | | | | |
| Volatility | −0.0756 | 0.6666 | 0.2461 | 0.2924 | 0.2429 | 0.7044 |
| Covariance | −0.6831 | 0.711 | 13.0173 | 6.7604 | 14.3894 | 39.1354 |
| Beta | −0.0021 | 0.0022 | 0.0135 | −0.0692 | 0.0177 | 0.0185 |
| Risk premium | −0.0055 | 0.0057 | 0.1132 | 0.0463 | 0.1243 | 0.3058 |
| Risk premium (%) | −0.3408 | 0.3547 | 7.0111 | 2.8672 | 7.6990 | 18.9396 |

This table shows the change in volatility, covariance, beta, and risk premium (absolute and percentage) for a typical shock in market and/or portfolio returns. The "joint shock" columns trace out the effect of market and portfolio shocks of the same sign. The starting point before the shocks occur is that of the sample mean for the variances, covariances, and leverage ratios. The typical shock is the mean of the absolute residual from the estimated model. The price of risk is the unconditional one estimated to be 1.896. All changes are annualized in percentage terms.

differentiate between positive and negative shocks, but we also look at the combined effect on portfolio characteristics of typical shocks occurring simultaneously at the market and portfolio level. We only look at shocks that are of the same sign at the market and portfolio level. Given that the unconditional betas are close to 1 for the medium and high leverage portfolios and about 0.6 for the low leverage portfolio, shocks of the opposite sign occur much less frequently than shocks of the same sign. The starting point before the shocks occur is at the sample mean for the variances, covariances, and leverage ratios.

Let us first focus on volatility. We report the difference between the annualized volatilities corresponding to the variances after and before the shocks. Portfolio shocks generate strong asymmetry for the low leverage portfolio, but the total volatility asymmetry remains less pronounced for this portfolio. Still, volatility increases by 45 basis points (bp) more in response to joint negative shocks compared to joint positive shocks. For the high (medium) leverage portfolio, joint positive shocks increase volatility by about 60 bp (33 bp), while joint negative shocks increase volatility by about 1.56% (1.86%). Note that the effects of the firm and market shocks do not add up to the joint effect because of the interaction effects in the model.

However, volatility is not priced in this model, covariances are. We multiply the covariance response by 52 to obtain the annualized value. We then have to multiply them by the leverage adjusted price of risk to obtain expected excess returns on the various portfolios. The covariance asymmetry phenomenon now emerges very clearly. First of all, portfolio shocks typically generate strong covariance asymmetry for all portfolios (assuming market shocks are zero). Second, these effects are rather small compared to the responses the market shocks generate. We find very strong covariance asymmetry for the medium and high leverage portfolios and weaker asymmetry for the low leverage portfolio.

How significant these changes in risk are for expected returns critically depends on the price of risk. As we discussed in Section 2.2, the price of risk in a GARCH-in-mean model is often difficult to estimate accurately. As an alternative measure, we computed the unconditional price of risk to be $Y^U = 1.896$. Table 8 reports absolute risk premium changes using the unconditional price of risk. For typical joint market and portfolio shocks, these changes vary between 12 and 55 bp (on an annualized basis). For our sample of the Nikkei index, the mean excess return is about 1.73% so that these changes in the risk premium are clearly economically significant.

Of course, the magnitude of the price of risk does not affect the asymmetric nature of the risk premium response to return shocks. The table illustrates clearly that negative return shocks increase the risk premium much more than positive shocks. It is in fact more intuitive to focus on percentage changes in the risk premium, which are not affected by the price of risk. In percentage terms, increased volatility drives up the market expected return by about 16% when caused by negative news, but only by about 5% when caused by positive news. When the increased uncertainty is priced and covariances increase, joint positive shocks generate percentage changes in risk premium of about 5% to 8%, whereas joint negative shocks lead to percentage changes in expected returns of more than 17%. The equilibrium interpretation is that volatility feedback induces return shocks that are exactly large enough to be consistent with the required change in expected return. When we normalize the shocks of different sign to be of the same magnitude, the underlying change in required return will be larger for negative shocks.

The time variation in risk premiums relative to its mean is quite considerable. Using the estimated price of risk, the mean risk premium on the market portfolio is only 0.104% with a standard deviation of

0.111%. The relatively small size of the risk premium reflects the fact that Japanese stocks appreciated little over our sample period because of the dramatic downturn in the overall market in the early 1990s. When we use the unconditional price of risk, the mean risk premium is 3.425% with a standard deviation of 3.642%. Since the price of risk is just a scale factor, Figure 6 plots the time-varying risk premium for the various portfolios with the price of risk put equal to one. It is apparent that the displayed time variation in the risk premium may be hard to capture in the context of a representative agent model.

Finally, Table 8 also reports the effects on beta. Negative numbers must mean that variances of the market portfolio rise more than the covariances. It is not surprising to see predominantly negative responses to a pure market shock, since we assume portfolio shocks to be zero. When both market and portfolio shocks are accounted for, there is no evidence for asymmetry in beta, except very weakly, for the medium leverage portfolio.

To complement the analysis in Table 8, we propose "economic impact curves," which are shown in Figure 7. These curves depict the effects implied by our model of a set of economically meaningful shocks on variance (panel A), covariance (panel B), beta (panel C), and risk premium (panel D). The effects are shown for all three portfolios

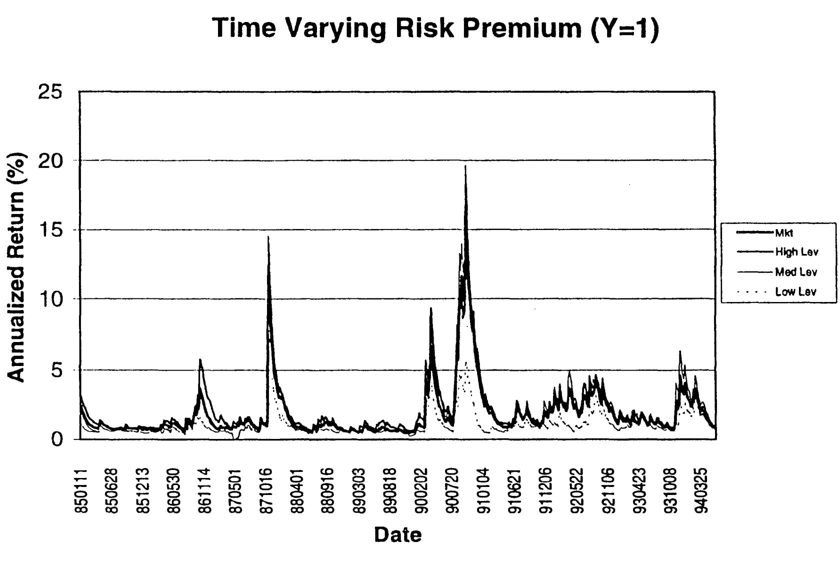


**Figure 6**
**Estimated time-varying risk premium**
This figure plots the risk premiums for the market portfolio and the three leverage portfolios implied by the model but sets the price of risk equal to one.

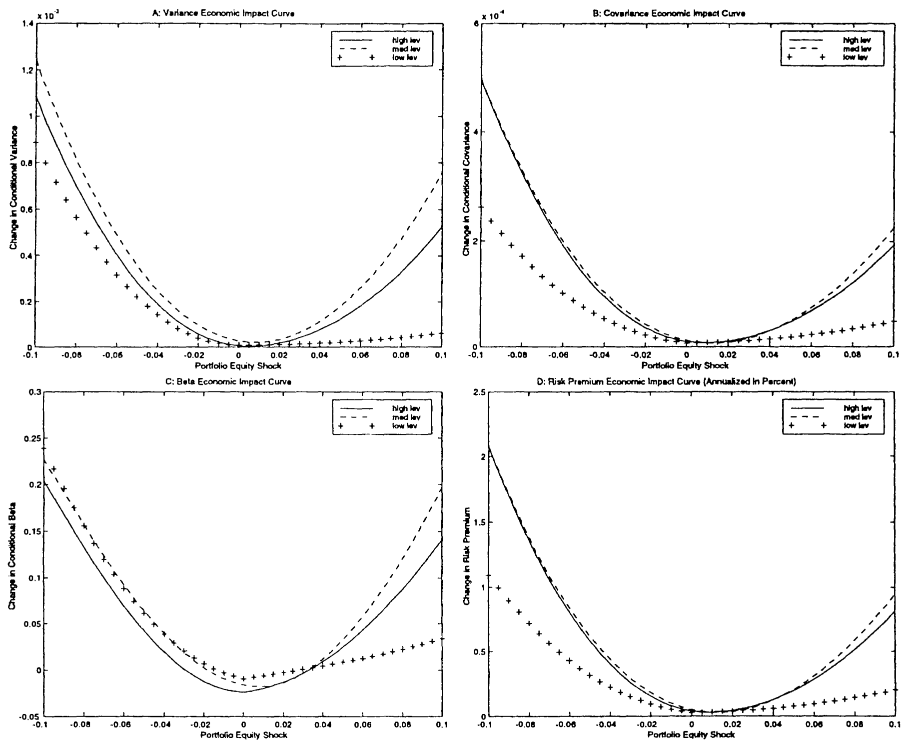


**Figure 7**
**Economic impact curves**
These curves depict the effects implied by our model of a set of economically meaningful shocks on variance (panel A), covariance (panel B), beta (panel C), and risk premium (panel D). The effects are shown for all three portfolios simultaneously. For each fixed portfolio shock, the figures integrate out the market shocks using the actual joint density of the shocks.

simultaneously. These graphs are much more informative than the impact surfaces for two main reasons. First, whereas we investigate the effects generated by a range of firm-level shocks, we graph them as a function of the corresponding equity shocks, using the average leverage ratios to make the transformation. This is important since our three equity portfolios, despite very different leverage ratios, have quite similar equity return volatilities (see Table 2). This is of course not surprising, given that firms with lower fundamental asset variances may optimally choose to be more highly levered (e.g., because the probability of distress is lower). Hence the size of the firm shocks underlying Figure 7 is very different for the three portfolios. In particular, the firm shocks are larger the smaller the average leverage ratio (see Table 2 for the relative magnitudes).

Second, news impact surfaces may mislead the eye by drawing attention to shock regions which have very low probability of actually occurring. To rectify this problem, Figure 7 integrates out the market shocks using the actual joint density of the shocks. More specifically, consider the impact curve for the high leverage portfolio. Define $\bar{\epsilon}_{kj} = [\bar{\epsilon}_{Mk}, \bar{\epsilon}_{Hj}, 0, 0]'$; it is the shock vector (at the firm level) for $\bar{\epsilon}_{Mk}$ and $\bar{\epsilon}_{Hj}$ with the other shocks put to zero. For each $\bar{\epsilon}_{Hj}$, we vary market shocks from $\bar{\epsilon}_M = -0.1$ to $\bar{\epsilon}_M = 0.1$ with a step size of 0.0005. This yields a grid of 400 possible market shocks for each $\bar{\epsilon}_{Hj}$. Let us generally define an impact (at the equity level) of a particular shock vector (which is the impact that would be graphed in a news impact surface) as $I(\bar{\epsilon}_{kj})$. Figure 7 graphs $\Sigma_{k=1}^{400} w(k) I(\bar{\epsilon}_{kj})$ for each $\bar{\epsilon}_{Hj}$, where $w(k) = f(\bar{\epsilon}_{kj}) / \Sigma_{l=1}^{400} f(\bar{\epsilon}_{lj})$ and $f(.)$ is the joint density of the shocks. The covariance matrix of the shocks we use to evaluate the density is the sample covariance matrix of the residuals. The resulting weight functions are very reasonable, putting more weight on small and "same sign" shocks. Please note that the impact curves do not necessarily start at the origin when portfolio shocks are zero since market shocks are not zero but rather can take on a range of shock levels for each portfolio shock level.

Figure 7 generally confirms our previous findings. In panel A, the sharp volatility asymmetry is now very apparent for all portfolios but seems more dramatic for the low leverage portfolio. The reason is that the firm shocks are quite large for this portfolio. To help interpret the numbers, the variance value of $1 \times 10^{-3}$ corresponds to an annualized volatility of 22.8%. The covariance asymmetry in panel B shows that covariances for the high and medium leverage portfolios increase by about twice as much for extreme negative shocks than they do for extreme positive shocks. The asymmetry for the low leverage portfolio looks more pronounced, but as with volatility, the absolute responses are considerably smaller compared to the other portfolios. This translates into smaller risk premiums in panel D that never exceed 1%, even for rather large shocks. For the other portfolios, the risk premium increase is less than 1% for positive shocks but can be up to 2% for extreme negative shocks.

The main message of Figure 7 is that for similar equity shocks, the low leverage portfolio exhibits much stronger asymmetry in volatility and risk. Although this seems counterintuitive relative to the standard leverage story, it is entirely consistent with the main results in this article: the volatility feedback effect dominates the leverage effect. That is, once we look at firm shocks there remains significant asymmetry. The reason that this asymmetry shows up so clearly for the low leverage portfolio in this graph is because the underlying firm shocks for the low leverage portfolio shocks are much larger than for the other portfolio shocks. Leverage effects themselves play a relatively minor role in generating the pattern we observe.

Panel C confirms the weakness of beta asymmetry for the high and medium leverage portfolios. It is clear that the strange, negative impact regions on the news impact surfaces (see Figure 5) mostly reflect low probability events. Of interest, since the unconditional beta of the low leverage portfolio is about 0.62 (see Table 2), its beta rises to about 0.9 with highly negative shocks, but hardly moves with highly positive shocks. The main reason that the beta dynamics look so different for the low leverage portfolio is that shocks are more predominantly idiosyncratic. Hence the strongly positive beta effects we see in Figure 5C when both market and portfolio shocks are positive do not dominate the average effect we show here.

## 4. Robustness of the Results

In this section we apply our model to a larger set of portfolios. Although primarily a robustness exercise, we also verify whether the findings of Cheung and Ng (1992) carry over to Japanese stocks. Cheung and Ng (1992) show that volatility asymmetry is much stronger for small U.S. stocks. To detect a potential size effect, we divide our universe of Japanese stocks into three groups (tertiles) based on average market capitalization over the sample period. Within the largest and smallest tertile, we select three leverage portfolios of five stocks. In particular, we rank the stocks by their average leverage ratios within the group and select these stocks with average leverage ratios closest to the leverage ratio of the portfolios in the main estimation. We do not exclude the orginal stocks, but only 6 of them are selected again among the 30 stocks in the 6 new portfolios. We are able to match the average leverage ratios of the portfolios in the main estimation rather closely, indicating that leverage ratios and size are not highly correlated. We then simply reestimate the model for the two resulting multivariate systems, constraining the price of risk to equal its previously estimated value.

To conserve space, we do not report all the results but summarize the main findings. First of all, both models remain well specified. We note only two rejections at the 1% significance level: the MEAN test rejects at the 0.4% level for the small stocks system, driven by a strong rejection for the low leverage portfolio. The CAPM2 test rejects at the 5% level for both systems.

Second, the main statistical conclusions we drew before remain valid. Likelihood ratio tests continue to reject, at all standard significance levels, the Christie model, the absence of GARCH and the symmetric GARCH model. We also strongly reject the hypothesis $d_{22} = d_{33} = d_{44} = 0$ for the small stock system, but the $p$-value is 1.3% for the large stock system, where market shocks are relatively more dominant. We conclude that for both systems, strong volatility asymmetry is still present once the leverage effect is filtered out.

Third, on an economic level we reported the presence of strong covariance asymmetry (with direct consequences for risk premium changes) for all portfolios, and the absence of a "leverage effect" in betas. These results largely continue to hold. Most of the covariance impact surfaces look very similar to the surfaces of their counterpart portfolios presented before. To demonstrate the similarities in an economically meaningful way, consider Table 9. Table 9 repeats the economic impact exercise of Table 8 for the large and small stock portfolios, but we restrict attention to volatilities, covariances, and betas. Generally there is strong covariance asymmetry for all portfolios, coming from both firm and market shocks. Covariance asymmetry is strongest for the small stock portfolios. We also confirm that most of the covariance asymmetry is due to market shocks, as was true in Table 8. For large stocks, we find either no beta asymmetry (high leverage portfolio) or beta asymmetry that is inconsistent with the leverage effect. The high and medium leverage portfolios in the small stock system show considerable beta asymmetry that is primarily driven by interaction shocks. In Table 8, on the other hand, there was either no (large and low leverage portfolios) or very weak (medium leverage portfolios) beta asymmetry. Hence we continue to find consistent evidence of covariance asymmetry across all portfolios, but not of beta asymmetry.

Fourth, whereas our major results appear qualitatively robust, we do not fully confirm the Cheung and Ng results for our sample. For example, asymmetries for the high leverage portfolio are stronger for the small-firm portfolio at the portfolio shock level but not at the market shock level. Nevertheless, when restricting attention to the joint

**Table 9**
**The economic effects of typical market and portfolio shocks for large and small size firms**

| | Firm shock | | Market shock | | Joint shock | |
|---|---|---|---|---|---|---|
| Panel A: Large-size firms | + | − | + | − | + | − |
| High leverage portfolio | | | | | | |
| Volatility | 2.4745 | 2.8895 | 0.0456 | 0.4995 | 1.9371 | 3.3693 |
| Covariance | −0.2707 | 0.2741 | −3.1976 | 13.2450 | 19.8201 | 52.0908 |
| Beta | −0.0036 | 0.0037 | −0.1267 | −0.1589 | 0.1447 | 0.1697 |
| Medium leverage portfolio | | | | | | |
| Volatility | 0.4105 | 0.5678 | 0.0067 | 0.6072 | 0.5265 | 1.3786 |
| Covariance | −0.4717 | 0.4840 | 0.9646 | 24.0373 | 12.1742 | 38.8979 |
| Beta | −0.0063 | 0.0065 | −0.1245 | −0.2103 | 0.0076 | −0.0846 |
| Low leverage portfolio | | | | | | |
| Volatility | 0.1664 | 0.5227 | 0.0136 | 1.0143 | 0.2875 | 0.6619 |
| Covariance | −0.3288 | 0.3402 | 1.7481 | 35.8314 | 10.4484 | 29.1965 |
| Beta | −0.0044 | 0.0046 | −0.1440 | −0.1978 | −0.0414 | −0.2539 |
| Panel B: Small-size firms | | | | | | |
| High leverage portfolio | | | | | | |
| Volatility | 4.2790 | 5.7166 | 0.1616 | 0.4196 | 3.0858 | 5.7265 |
| Covariance | −0.2316 | 0.2351 | −4.5847 | 9.5651 | 20.4013 | 73.8355 |
| Beta | −0.0028 | 0.0028 | −0.0863 | −0.1166 | 0.1956 | 0.3713 |
| Medium leverage portfolio | | | | | | |
| Volatility | 1.4589 | 2.7283 | 0.2554 | 0.4998 | 0.4728 | 2.9338 |
| Covariance | −0.2560 | 0.2649 | −8.0628 | 11.3326 | 11.7415 | 72.3693 |
| Beta | −0.0031 | 0.0032 | −0.1194 | −0.0697 | 0.1040 | 0.3936 |
| Low leverage portfolio | | | | | | |
| Volatility | 0.3716 | 1.2500 | 0.0003 | 0.2860 | 0.3956 | 2.3701 |
| Covariance | −0.5609 | 0.5849 | −0.2614 | 22.3891 | 10.3778 | 68.3277 |
| Beta | −0.0068 | 0.0071 | −0.0970 | −0.3448 | 0.0231 | 0.0040 |

This table shows, for a large- and small-firm portfolios, the change in volatility, covariance, and beta for a typical shock in market and/or portfolio returns. The "joint shock" columns trace out the effect of market and portfolio shocks of the same sign. The starting point before the shocks occur is that of the sample mean for the variances, covariances, and leverage ratios. The typical shock is the mean of the absolute residual from the estimated model. The price of risk is the unconditional one estimated to be 1.896. All changes are annualized in percentage terms.

shocks in Table 9, it appears that small firms show somewhat stronger asymmetry. As we indicated above, they also show stronger covariance asymmetry. This does not necessarily prove that size is the cause of the stronger asymmetries, since we have not controlled for other potential factors influencing asymmetry. For example, it may be that small firms have generally high betas and that the more pronounced asymmetry found for small firms really reflects the stronger volatility feedback effects all high beta firms display. We do not have enough beta dispersion across portfolios to formally examine this, but there are some rough indications. Unlike the other portfolios, the volatility dynamics of the low leverage portfolio in our original estimation are primarily driven by portfolio shocks and the leverage effect remains relatively important. That portfolio also has a distinctly lower beta than the other two portfolios. From the six portfolios we examine in our robustness check, the lowest beta portfolio is the small size high leverage portfolio. It is striking that from all the portfolios examined, it shares with the low leverage portfolio the property that the coefficient on $\eta_{i,t-1}$ is significantly larger than the coefficient on $\eta_{M,t-1}$, although this does not translate in substantially different economic effects.

## 5. Conclusions

In this article we investigate the leverage effect and the time-varying risk premium explanations of the asymmetric volatility phenomenon at both the market and firm level. We propose a conditional CAPM model with a GARCH-in-mean parameterization ensuring time variation in conditional means, variances, and covariances. By assuming riskless debt and formulating the CAPM at the firm level, stock returns potentially exhibit leverage effects both through changes in actual leverage ratios and through changes in firm-level volatilities. Of importance, conditional covariances also exhibit leverage and asymmetry effects.

We apply the model to the market portfolio and three portfolios, grouped by leverage, constructed from Nikkei 225 stocks. The asymmetric patterns we find are primarily driven by the variance dynamics at the firm level and not by changes in leverage. Our model nests the Christie model under riskless debt and the model is rejected. Our specification tests indicate that it is unlikely that relaxing the riskless debt assumption will salvage the leverage explanation. To see more clearly the importance of firm variance dynamics, consider Figure 8. There, we graphed three volatility series for the market and the three leverage portfolios. One is the actual conditional volatility implied by the model; one sets firm volatility equal to its sample average (the Christie model) and one sets the leverage ratio at its sample average (firm volatility dynamics without leverage effects). The graphs very clearly show how firm volatility dynamics dominate the temporal behavior of volatility. The Christie model would generate volatilities that are too smooth. Nevertheless, as predicted by Christie's (1982) analysis, for the medium and in particular for the high leverage portfolios, ignoring variation in leverage may yield very different volatility estimates. The leverage effect on volatility seems small compared to the asymmetry generated through the shocks in the GARCH specification. The peaks in the volatility graphs typically correspond to large declines in the market. When such major market movements occur, all portfolios react similarly (betas go to 1) and large increases in volatility occur. This is a clear illustration of the volatility feedback mechanism generating volatility asymmetry.

The main mechanism behind the asymmetry for the high and the medium leverage portfolios is covariance asymmetry. Negative shocks increase conditional covariances substantially, whereas positive shocks have a mixed impact on conditional covariances. As the graph indicates, only a small part of this phenomenon can be attributed to a pure

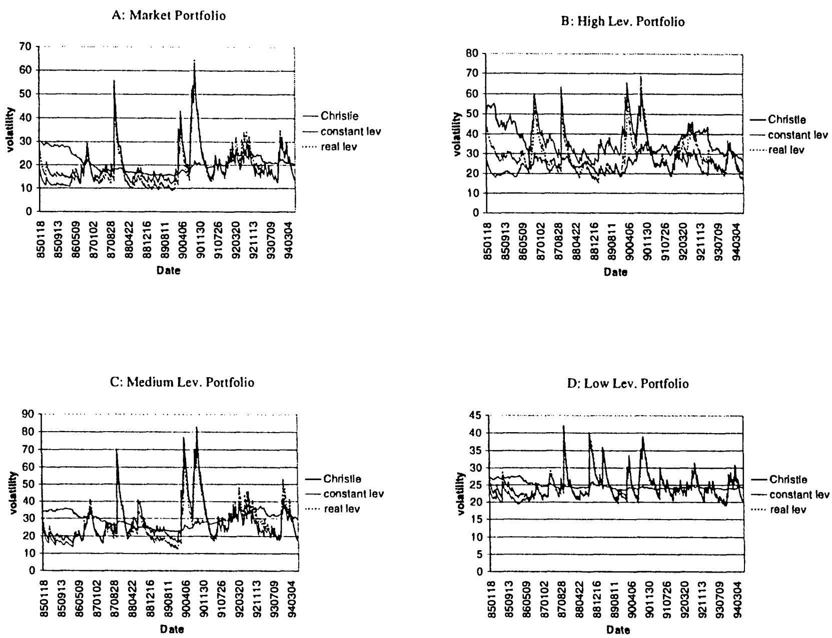


**Figure 8**
**Conditional volatility implied by models**
This figure graphs three volatility series in annualized percentage for the market (panel A) and the three leverage portfolios (panels B, C, and D). One is the actual conditional volatility implied by the model; one sets firm volatility equal to its sample average (the Christie model) and one sets the leverage ratio at its sample average (firm volatility dynamics without leverage effects).

leverage effect. The conditional betas do not behave as predicted by the leverage story, except for the low leverage portfolio. Taken together our results suggest that "the leverage effect" may be a misnomer.

Although our results seem consistent with the existence of time-varying risk premiums and volatility feedback, there may be other factors driving the results. For example, it is unlikely that standard general equilibrium models with an expected-utility maximizing representative agent would generate time variation in equity risk premiums that is as large as shown in Figure 6. The challenge for future research will nonetheless be to endogenize the volatility feedback mechanism as is attempted in Wu (1998).

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