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Scaling and correlation in financial time series

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Abstract

We discuss the results of three recent phenomenological studies focussed on understanding the distinctive statistical properties of financial time series – (i) *The probability distribution of stock price fluctuations*: Stock price fluctuations occur in all magnitudes, in analogy to earthquakes – from tiny fluctuations to very drastic events, such as the crash of 19 October 1987, sometimes referred to as "Black Monday". The distribution of price fluctuations decays with a power-law tail well outside the Lévy stable regime and describes fluctuations that differ by as much as 8 orders of magnitude. In addition, this distribution preserves its functional form for fluctuations on time scales that differ by 3 orders of magnitude, from 1 min up to approximately 10 days. (ii) *Correlations in financial time series*: While price fluctuations themselves have rapidly decaying correlations, the magnitude of fluctuations measured by either the absolute value or the square of the price fluctuations has correlations that decay as a power-law, persisting for several months. (iii) *Volatility and trading activity*: We quantify the relation between trading activity – measured by the number of transactions $N_{\Delta t}$ – and the price change $G_{\Delta t}$ for a given stock, over a time interval $[t,\ t+\Delta t]$. We find that $N_{\Delta t}$ displays long-range power-law correlations in time, which leads to the interpretation that the long-range correlations previously found for $|G_{\Delta t}|$ are connected to those of $N_{\Delta t}$. © 2000 Elsevier Science B.V. All rights reserved.

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0. Introduction

The distinctive statistical properties of financial time series are increasingly attracting the interest of statistical physicists, both from the point of view of data analysis and modeling [1–62]. Apart from its practical importance and its importance in

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modern economics, the scientific interest in studying financial markets stems from the fact that there is a wealth of data available for financial markets which makes it arguably the one complex system most amenable to quantification and ultimately scientific understanding. In addition, it is also possible that the dynamics underlying financial markets are "universal" as exemplified in several studies [63,64] that have noted the statistical similarity of the properties of observables across quite different markets. Moreover, a precise statistical description of price movements is important in practical applications such as Value-at-Risk estimations and derivative pricing [65–73].

Several recent studies attempt to uncover and explain the peculiar statistical properties of financial time series such as stock prices, stock market indices or currency exchange rates. This talk reviews recent results on (a) the distribution of stock price fluctuations and its scaling properties, (b) time-correlations in financial time series, and (c) relation between price fluctuations and intensity of trading.

1. Distribution of price fluctuations

The nature of the distribution of price fluctuations in financial time series is a long standing open problem in finance which dates back to the turn of the century. In 1900, Bachelier proposed the first model for the stochastic process of returns – an uncorrelated random walk with independent, identically Gaussian distributed (*i.i.d*) random variables [1]. This model is natural if one considers the return over a time scale Δt to be the result of many independent "shocks", which then lead by the central limit theorem to a Gaussian distribution of returns [1]. However, empirical studies [4,37–40] show that the distribution of returns has pronounced tails in striking contrast to that of a Gaussian. Despite this empirical fact, the Gaussian assumption for the distribution of returns is widely used in theoretical finance because of the simplifications it provides in analytical calculation; indeed, it is one of the assumptions used in the classic Black–Scholes option pricing technique [74].

In his pioneering analysis of cotton prices, Mandelbrot observed that in addition to being non-Gaussian, the process of returns shows another interesting property: "time scaling" – that is, the distributions of returns for various choices of Δt, ranging from 1 day up to 1 month have similar functional forms [4]. Motivated by (i) pronounced tails, and (ii) a stable functional form for different time scales, Mandelbrot [4] proposed that the distribution of returns is consistent with a Lévy stable distribution [2,3].

Recent studies [75–79] on considerably larger time series using larger databases show quite different asymptotic behavior for the distribution of returns. Our recent work [75] analyzed three different data bases covering securities from the three major US stock markets. In total, we analyzed approximately 40 million records of stock prices sampled at 5 min intervals for the 1000 leading US stocks for the 2-year period 1994–1995 and 35 million daily records for 16,000 US stocks for the 35-year period

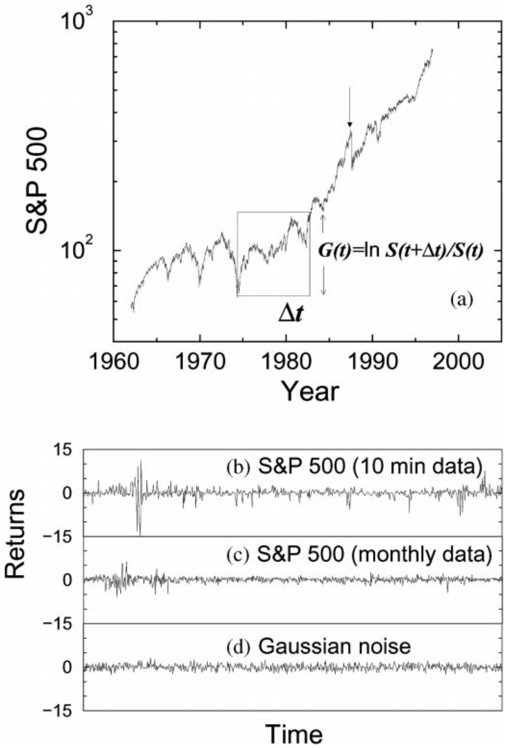


Fig. 1. (a) The daily records of the S&P 500 index for the 35-year period 1962–1996 on a linear-log scale. Note the large jump which occurred during the market crash of October 19, 1987. Sequence of (b) 10 min returns and (c) 1 month returns of the S&P 500 index, normalized to unit variance. (d) Sequence of *i.i.d.* Gaussian random variables with unit variance, which was proposed by Bachelier as a model for stock returns [1]. For all three panels, there are 850 events – i.e., in panel (b) 850 min and in panel (c) 850 months. Note that, in contrast to (b) and (c), there are no large events in (d).

1962–1996. We study the probability distribution of returns (Fig. 1a–c) for individual stocks over a time interval Δt, where Δt varies approximately over a factor of 10^4 – from 1 min up to more than 1 month. We also conduct a parallel study of the S&P 500 index.

Our key finding is that the *cumulative* distribution of returns for both individual companies (Fig. 2a) and the S&P 500 index (Fig. 2b) can be well described by a power-law asymptotic behavior, characterized by an exponent $\alpha \approx 3$, well outside the stable Lévy regime $0 < \alpha < 2$. Further, it is found that the distribution, although not a stable distribution, retains its functional form for time scales up to approximately 16 days for individual stocks and approximately 4 days for the S&P 500 index (Fig. 2a). For larger time scales our results are consistent with break-down of scaling behavior, i.e., convergence to Gaussian [75]. Similar results have also been found for currency exchange data [79].

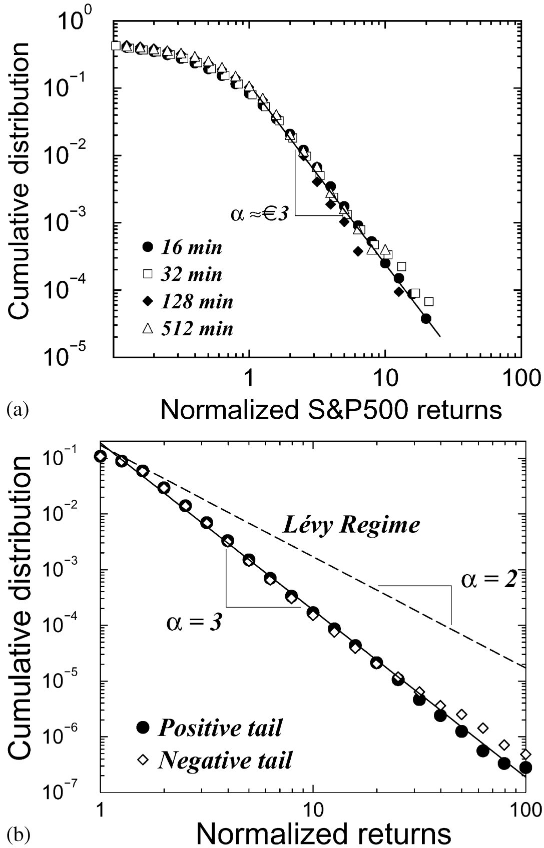
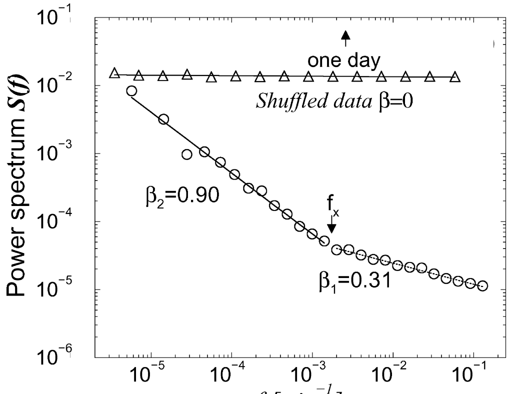


Fig. 2. (a) Log–log plot of the cumulative distribution of normalized returns of the S&P 500 index. The positive tails are shown for $\Delta t = 16, 32, 128, 512$ mins. Power-law regression fits yield estimates of the asymptotic power-law exponent $\alpha = 2.69 \pm 0.04$, $\alpha = 2.53 \pm 0.06$, $\alpha = 2.83 \pm 0.18$ and $\alpha = 3.39 \pm 0.03$ for $\Delta t = 16, 32, 128$ and 512 mins, respectively. (b) The positive and negative tails of the cumulative distribution of the normalized returns of the 1000 largest companies in the TAQ database for the 2-year period 1994–1995. The solid line is a power-law regression fit in the region $2 \leqslant x \leqslant 80$.

2. Time correlations in price fluctuations

In addition to the probability distribution, an aspect of equal importance for the characterization of any stochastic process is the quantification of correlations. Studies of the autocorrelation function of the returns show exponential decay with characteristic decay times of only 4 min [80] consistent with the efficient market hypothesis [81]. This is paradoxical, for in the previous section, we have seen that the distribution of returns, in spite of being a non-stable distribution, preserves its shape for a wide range of Δt. Hence, there has to be some sort of correlations or dependencies that prevent the central limit theorem to take over sooner and preserve the scaling behavior.



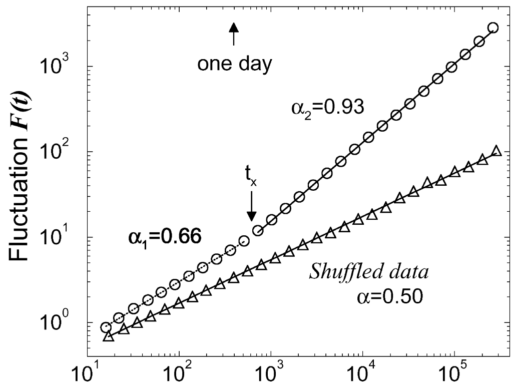


Fig. 3. Plot of (a) the power spectrum $S(f)$ and (b) the detrended fluctuation analysis $F(t)$ of the absolute values of returns $g(t)$, after detrending the daily pattern [82,83] with the sampling time interval $\Delta t = 1$ min. The lines show the best power-law fits (R values are better than 0.99) above and below the crossover frequency of $f_\times = (\frac{1}{570})\,\mathrm{min}^{-1}$ in (a) and of the crossover time, $t_\times = 600$ min in (b). The triangles show the power spectrum and DFA results for the "control", i.e., shuffled data.

Indeed, lack of linear correlation does not imply independent returns, since there may exist higher-order correlations. Recently, Liu and his collaborators found that the amplitude of the returns, the absolute value or the square – closely related to what is referred to in economics as the *volatility* [84–88] – shows long-range correlations [42–46,82,83,89–91] with persistence [92] up to several months, Fig. 3a and b. They analyzed the correlations in the absolute value of the returns [82,83] of the S&P 500 index using traditional correlation function estimates, power spectrum and the recently developed detrended fluctuation analysis (DFA). All the three methods show the existence of power-law correlations with a cross-over at approximately 1.5 days. For the S&P 500 index, DFA estimates for the exponents characterizing the power-law correlations are $\alpha_1 = 0.66$ for short time scales smaller than ≈ 1.5 days and $\alpha_2 = 0.93$ for longer time scales up to a year (Fig. 3b). For individual companies, the same methods yield $\alpha_1 = 0.60$ and $\alpha_2 = 0.74$, respectively. The power spectrum gives consistent estimates of the two power-law exponents (Fig. 3a).

The long memory in the amplitude of returns suggests that it is useful to define another process, referred to as the volatility. Volatility of a certain stock measures how much it is likely to fluctuate. It can also be related to the amount of information arriving at any time. The volatility can be estimated for example by the local average of the absolute values or the squares of the returns. In their recent work on the statistical properties of volatility Liu et al. [83,91] show that the volatility correlations show asymptotic $1/f$ behavior [82,83,91]. Using the same data bases as above, Liu and his collaborators also study the *cumulative* distribution of volatility [82,91] and find that it is consistent with a power-law asymptotic behavior, characterized by an exponent $\mu \approx 3$, just the same as that for the distribution of returns. For individual companies also, one finds a similar power-law asymptotic behavior [83]. In addition, it is also found that the volatility distribution scales for a range of time intervals just as the distribution of returns.

3. Possible approaches

We have looked mainly at two empirical results: (i) the distribution of fluctuations, which shows a power-law behavior well outside the stable Lévy regime, and yet preserves its shape – scales – for a range of time scales and (ii) the long-range correlations in the amplitude of price fluctuations. How are the two results related?

Previous explanations of scaling relied on Lévy stable [4] and exponentially-truncated Lévy processes [5,37]. However, the empirical data that we analyze are not consistent with either of these two processes. In order to confirm that the scaling is *not* due to a stable distribution, one can randomize the time series of 1 min returns, thereby creating a new time series which contains *statistically independent* returns. By adding up n consecutive returns of the shuffled series, one can construct the n min returns. Both the distribution and its moments show a rapid convergence to Gaussian behavior with increasing n, showing that the time dependencies, specifically volatility correlations, are intimately connected to the observed scaling behavior [75].

Using the statistical properties summarized above, can we attempt to deduce a statistical description of the process which gives rise to this output? Let us first focus on the observed long-range correlations in $|G|$. One can express $G = \mathrm{sgn}(G)|G|$. The fact that G has only short-range correlations implies $\mathrm{sgn}(G)$ is uncorrelated. This can be expressed more generally in the form $G = \varepsilon V$, where ε is an uncorrelated variable with some distribution, and V is the instantaneous standard deviation, often called volatility (this hypothesis is often called the stochastic volatility hypothesis). Note that the reason to consider V as a variable in its own right comes from the empirical fact that estimated local variances seem to fluctuate significantly with time, and from the observation that $|G|$ is long-range correlated. In order to account for time dependencies in V, one can either postulate a deterministic dependence of V on past values of V or G^2

which leads one to ARCH [63,93] class of models. However it assumes finite memory of past events and hence is not consistent with long-range correlations in volatility. A consistent statistical description may involve extending the traditional ARCH model to include long-range volatility correlations [94]. The alternative would be to treat V as a stochastic variable, which leads to stochastic volatility models.

How can we physicists approach this problem? One approach to understand the mysterious statistical features of price fluctuations is in the spirit of Bachelier who developed Gaussian diffusion description of price movements, and ask where the Gaussian description went wrong. Bachelier's model was to consider price changes G in a time interval Δt as being composed out of several changes δp_i, which can be effectively considered as occurring in continuous time. In other words

$$G \equiv \sum_{i=1}^{N_{\Delta t}} \delta p_i \,, \tag{1}$$

where $N_{\Delta t}$ is the number of transactions in Δt. If $N_{\Delta t} \gg 1$, and δp_i have finite variance, then one can apply the classic version of the central limit theorem, whereby one would obtain $P(G)$ as Gaussian. It is implicitly assumed in this description that $N_{\Delta t}$ is not varying too much, i.e., $N_{\Delta t}$ has only Gaussian fluctuations around a mean value. Let us start by asking to what extent this is true.

In a typical day, there might be as many as $N_{\Delta t} = 1000$ trades for an actively traded stock. Fig. 4a shows the time series of $N_{\Delta t}$ for an actively traded stock sampled at 15 min intervals contrasted with a series of Gaussian random numbers. From the presence of several events of the magnitude of tens of standard deviations, it is apparent that $N_{\Delta t}$ is distinctly non-Gaussian [95–104]. Let us first quantify the statistics of $N_{\Delta t}$. We first analyze the distribution of $N_{\Delta t}$. Fig. 3b shows that $P(N_{\Delta t})$ decays as a power-law

$$P(N_{\Delta t}) \sim N_{\Delta t}^{-(1+\beta)} \,, \tag{2}$$

where $\beta \approx 3.5$ for five actively traded stocks. A more extensive analysis on 1000 stocks [104] gives values of β around the average value $\beta = 3.4$. Thus $N_{\Delta t}$ behaves in a remarkably non-Gaussian manner.

We also analyze correlations in $N_{\Delta t}$ Instead of analyzing the correlation function directly, we use the method of detrended fluctuation analysis [105]. We plot the detrended fluctuation function $F(\tau)$ as a function of the time scale τ. Absence of long-range correlations would imply $F(\tau) \sim \tau^{0.5}$, whereas $F(\tau) \sim \tau^{\nu}$ with $0.5 < \nu \leqslant 1$ implies power-law decay of the correlation function,

$$\langle [Q_{\Delta t}(t)][Q_{\Delta t}(t+\tau)] \rangle \sim \tau^{-\nu_{cf}}, \quad [\nu_{cf} = 2 - 2\,\delta] \,. \tag{3}$$

We obtain the value $\nu \approx 0.85$ for the same five stocks as before (Fig. 5). On extending this analysis for a set of 1000 stocks we find the mean value $\nu_{cf} \approx 0.3$ [104].

It is possible to relate this to the correlations in $|G|$, which is related to the variance V^2 of G. From Eq. (1), we see that $V^2 \propto N_{\Delta t}$ under the assumption that δp_i are independent. Therefore, the long-range correlations in $N_{\Delta t}$ is one reason for the observed long-range correlations in $|G|$. In other words, highly volatile periods in the market

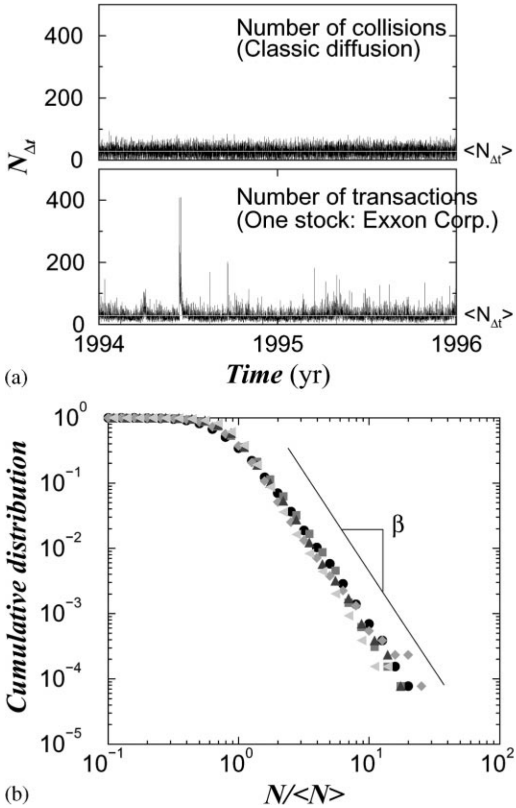


Fig. 4. Statistical properties of $N_{\Delta t}$. (a) The lower panel shows $N_{\Delta t}$ for Exxon Corporation with $\Delta t = 30$ min and the average value $\langle N_{\Delta t} \rangle \approx 52$. The upper panel shows a sequence of uncorrelated Gaussian random numbers with the same mean and variance, which depicts the number of collisions in $N_{\Delta t}$ for the classic diffusion problem. Note that in contrast to diffusion, $N_{\Delta t}$ for Exxon shows frequent large events of the magnitude of tens of standard deviations, which would be forbidden for Gaussian statistics. (b) The cumulative distribution of $N_{\Delta t}$ for five stocks: Exxon, General Electric, Coca Cola, AT&T, Merck show similar decay consistent with a power-law behavior with exponent $\beta \approx 3.4$.

persist due to the persistence of trading activity, that is in turn related to how news influences stock prices. Indeed, a remarkable consequence of our study is to quantify how price changes are related to $N_{\Delta t}$, which is connected to how news "drives" trading activity $N_{\Delta t}$. News comes in all magnitudes – from drastic "newsbreaks" to tiny pieces of information.

Could it be that the tail exponent β of the $P(N_{\Delta t})$ is connected to the exponent α of $P(G)$? We have seen that the distribution $P\{N_{\Delta t} > x\} \sim x^{-\beta}$ with $\beta \approx 3.4$ (Fig. 4). Therefore, $P\{\sqrt{N_{\Delta t}} > x\} \sim x^{-2\beta}$ with $2\beta \approx 6.8$. Therefore, $N_{\Delta t}$ alone cannot explain the value $\alpha \approx 3$. Instead, $\alpha \approx 3$ must arise from elsewhere. Upon examining the behavior of Eq. (1), we can see that in addition to G depending on $N_{\Delta t}$, it should also depend on $W^2_{\Delta t}$, the variances of the individual transaction changes δp_i. In fact, we can carry the analysis through to $W_{\Delta t}$ [104], whereby we find that the distribution of $W_{\Delta t}$, which

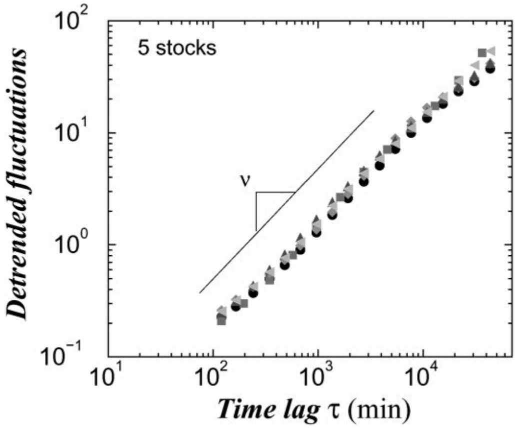


Fig. 5. Detrended fluctuation function $F(\tau)$ for the same five stocks as before. Regressions yield values of the slope $\nu \approx 0.85$, consistent with long-range correlations.

decays with approximately the same exponent $\gamma \approx \alpha \approx 3$. Thus the power-law tails in $P(G_{\Delta t})$ appear to originate from the power-law tail in $P(W_{\Delta t})$.

In sum, we have related volatility to two different microscopic quantities: (a) the transaction frequency, that is the number of transactions $N_{\Delta t}$ that occur in a time interval and (b) the "impact" of a transaction, measured by the variance $W_{\Delta t}^2$ of price changes due to all transactions, in a time interval. One can view this result using an analogy with classic diffusion, where the spread of an ink drop is determined by two microscopic quantities: (a) the collision frequency, that is the number of collisions $N_{\Delta t}$ that occur in a time interval and (b) the impact of collisions, measured by the variance $W_{\Delta t}^2$ of the displacements between collisions in that time interval. For stock prices, $N_{\Delta t}$ and $W_{\Delta t}$ behave remarkably differently from their analogs in classic diffusion. Thus, one could summarize by saying that price movements are equivalent to a complex variant of classic diffusion, where the price evolves through transactions in much the same way as an ink drop spreads through molecular collisions, not in a quiet container of water (as in classic diffusion), but rather in a bubbling hot spring, where the bubbling characteristics depend on a wide range of time and length scales.

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