Discrete Scaling in Stock Markets Before Crashes

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Abstract: We propose a picture of stock market crashes as critical points in a hierachical system with discrete scaling. The critical exponent is then complex, leading to log-periodic fluctuations in stock market indexes. We present "experimental" evidence in favor of this prediction. This picture is in the spirit of the known earthquake-stock market analogy and of recent work on log-periodic fluctuations associated with earthquakes.

The study of earthquakes as critical points has been of interest for some time now [1, 2, 3, 4, 5]. At a critical point one expects a scaling regime to set in. Recently it has been suggested [6, 7] that the underlying scale invariance is discrete, as expected for a hierarchical system. Then the critical exponent is complex and the scaling law near the critical point is "decorated" by log-periodic corrections ($\Re \tau^{\alpha+i\omega} = \tau^\alpha \cos(\omega \log \tau)$) Evidence for such log-periodic fluctuations was found [6] in measurements of the concentration of Cl^- and SO_4^{--} ions in mineral water collected over the 20 months immediately preceeding the 1995 Kobe earthquake at a source close to its epicenter. Similar evidence was also found [7] in the cumulative Benioff strain in connection with the 1989 Loma Prieta earthquake. It was proposed [6] that monitoring log-periodic fluctuations may ultimately prove useful in earthquake prediction.

Such fluctuations seem generic in hierarchically organized rupture processes. In the spirit of an earthquake-stock market analogy, this has led us to consider the possibility that log-periodic fluctuations may appear in stock market indices over a period preceding a crash. The stock market index (S&P 500, Dow-Jones, NIKKEI, ...) is to play here the same role as the Cl^- ion concentration played in the the analysis of the Kobe earthquake. Fortunately such indices are closely monitored and good data are plentiful. The scaling

variable is again time t. Call $c(t)$ the index as a function of time. Truncating at the first harmonic of a general log-periodic correction, we can then write for $c(t)$ the same formula as that given in [6] for the ion concentration

$$c(t) = A + B(t_c - t)^\alpha \left[1 + C \cos[\omega \log(t_c - t) + \varphi]\right]. \tag{1}$$

As a first test of this idea let us consider the crash which occured in New York on October 19, 1987. As the relevant index let us use the S&P 500, which dropped by more than 20% that day. In figure 1a we present a fit of the 1986-1987 weekly S&P 500 using Eq. (1) . One can see clearly two full periods of the log-periodic oscillation and some more oscillatory behavior close to the time of the crash. We only fit data up to three weeks before the crash, where the fit starts very fast oscillations. A reasonable fit is obtained this way with parameters given in Table 1 (where we omitted the parameters A, B and φ, which depend on the arbitrary normalization of the index or on the time scale).

Error bars of ± 10 were assigned to each data point for purposes of calculating χ^2. To a certain extent this is arbitrary, but it also reflects the possibility of higher harmonics neglected in our fit and of noise. This error assignment will be used in all other fits except the next one. Concerning the values of t_c for this and our other fits as well, the actual crash dates have been used as input. Given that here we used weekly, and in all other fits monthly, data, the small discrepancies between the crash dates in Table 1 (given there in the yy/mm/dd format) and the real crash dates are devoid of significance.

In figure 1b we fit a NY crash (defined here as a drop in the index by more than 10%) in 1962, using monthly S&P 500 data. This time the error bars used to calculate χ^2 were set at ± 2.5, since the S&P 500 was considerably lower in the Fifties than in the Eighties. In figure 1c the 1929 NY crash (using monthly Dow-Jones data) is fit. The parameters for all these fits are given in table 1. We also considered the 1990 Tokyo crash using *scanned* NIKKEI data and found similar log-periodic behavior, but we intend to refine this with tabulated data.

There is evidence for log-periodicity in these fits. While log-periodicity is indicative of an impending crash, one can easily think of crashes not associated with any log-periodic behavior, for instance those caused by sudden unexpected world events. We should add that we defined a crash as a change

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by 10% or more of an index over a short time interval (e.g 1 day in 1987). In all cases the change is negative (a crash) rather than positive (an upsurge).

Notice that all these fits range over time intervals of 2-8 years before the crash. By contrast, let us attempt to fit the S&P 500 for the time interval 1991-1994 during (and immediately after) which no crash occured. This is done in figure 1d and its parameters can be found in table 1. The parameter C which measures the relative importance of the log-periodic fluctuations is now two orders of magnitude smaller than in all previous fits and that there are therefore no detectable oscillations. The critical time t_c itself is in the *past*. Restricting to a shorter calm period, say 1992-1993, no significant oscillations are observed either. This further supports the discrete scaling picture advocated here.

Let us now return to the 1987 crash. We have fitted Eq. (1) to the 1986-1987 interval leading up to this crash. One might discern further periods of the log-periodic fluctuations over a longer time period. In fact if we select the interval 1980-1987, then six oscillation periods come into view. Can one fit these to equation (1)? The answer is yes and the corresponding best fit is given in Fig. 1e and the parameters again in table 1. There is a problem now, for the frequency ω is now quite different from that obtained from the 1986-1987 fit. One might think that relaxing the assumed constancy of the background parameter A might alleviate this problem. Yet assuming A to be a quadratic polynomial in time (at the expense of two added parameters) has no significant effect. The eight-year fit involves a higher frequency ω, which makes it overoscillate in the overlap region with the two-year fit, so that over the final years 1986 and 1987 a new complex critical exponent takes over.

The next phenomenological question concerns the accuracy with which the time t_c of the crash can be predicted from the monitoring of log-periodic fluctuations. This is an interesting question indeed and through much more detailed statistical study one could settle it for past crashes. The authors of ref. [6] have expressed optimism concerning the corresponding problem for earthquakes, namely predicting earthquakes on the basis of monitoring log-periodic fluctuations at the "right" sites. But seismic activity is a natural phenomenon impervious to human monitoring. By contrast, if in the future large groups of investors, who believe they have observed a pattern of log-periodic fluctuations in a stock market index, proceed on that basis to predict a crash time, they may find such a prediction both unprofitable and "counterproductive".

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As a rule, discrete scaling is connected with hierarchical models [7]. It is thus natural to invoke a hierarchical structure to account for the discrete scaling connected with log-periodic fluctuations in stock market indexes. A clear hierarchical structure is present among investors which range from the individual small investor to the largest mutual funds. At the other end, the stocks themselves arrange themselves in subsectors, sectors, industries, ... A fiber bundle-like model [5] exploiting these hierarchies can be envisaged and we hope to return to this subject elsewhere. Here we prefer to keep the discussion "phenomenological" and content ourselves with the above presentation of evidence for log-periodic fluctuations in stock market indexes .

While this work was being completed, we learned that D. Sornette and coworkers have also considered this problem with similar results.

References

[1] See e.g. Keilis-Borok V.I. Editor, Phys. Earth and Planet. Int. **61**, Nos. 1-2 (1990); further references are contained here, as well as in [2] and [3] below.

[2] Allègre C.J., Le Mouel J.L. and Provost A., Nature 297, 47 (1982).

[3] Smalley R.F., Turcotte D.L. and Solla S.A., J. Geophys. Res. 90, 1894 (1985).

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[5] Newman W.I., Gabrielov A., Durand T., Phoenix S.L. and Turcotte D.L., Physica **D77**, 200 (1994).

[6] Johansen A., Sornette D., Wakita H., Tsunogai U., Newman W.I. and Saleur H., preprint, May 1995, submitted to Nature.

[7] Saleur H., Sammis C.G. and Sornette D., preprint USC-95-02.

Table 1

years/index	figure	C	α	ω	t_c	χ^2/degrees freedom
86-87/S&P 500	1a	-0.035	0.2	8.06	87/10/19	45.41/76
53-62/S&P 500	1b	0.19	0.68	7.41	62/01/04	78.17/88
20-29/DJ	1c	-0.014	0.14	8.73	29/10/22	101/103
91-94/S&P 500	1d	0.00058	0.57	12.01	90/05/07	60.79/41
80-87/S&P 500	1e	-0.036	0.2	12.94	87/10/15	99.47/84

Figure Caption

Figure 1. Fits with Eq. (1) of the: a) 1986-1987 S&P 500 ; b) 1953-1962 S&P 500; c) 1920-1929 Dow-Jones; d) 1990-1994 S&P 500; e) 1980-1987 S&P 500. The values of the parameters and the χ^2 for each of these fits are given in Table 1.