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EVIDENCE ON THE PRESENCE AND CAUSES OF SERIAL CORRELATION IN MARKET MODEL RESIDUALS

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Studies of returns on common stocks have observed positive market index autocorrelation (see Fisher [10] and Dimson [6]), negative autocorrelation of market model residuals (see Fisher [10] and Fama, Fisher, Jensen, and Roll [9]), and a deterioration in the market model R^2 as the returns measurement period is shortened (see Pogue and Solnik [17], Altman, Jacquillat, and Levasseur [1], and Schwartz and Whitcomb [19]). We present further evidence on the strength of these findings and show that they are concurrent events. That is, common factors can explain both positive index and negative residual autocorrelation, and these correlation patterns in turn cause R^2 to fall as the differencing interval is shortened.

A primary purpose of this paper is to explain the rather broad patterns of residual autocorrelation in the data. To this end, we consider various technical and economic factors which could account for the autocorrelation we have observed. Our evidence suggests that the major cause is the "(Lawrence) Fisher effect," which we suggest encompasses the effect of both market thinness and delayed portfolio rebalancing.

Section I briefly surveys the evidence in the literature on returns autocorrelation and establishes that a negative R^2, differencing interval relationship can be explained by autocorrelation in residuals and in market index returns. Section II presents our findings on the systematic decrease in R^2 and our evidence on the autocorrelation structure in market model residuals. Various possible causal factors of such a pattern are then considered and tested in Section III. Section IV contains our concluding remarks.

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I. Previous Results and Design of Current Study

While the extensive random walk literature reports intertemporal dependence patterns in common stock returns,[1] the evidence is quite clear that these patterns are not strong enough to warrant rejection of the efficient markets hypothesis, at least for U.S. stock markets where they have been most extensively studied. Fama [7] reports first-order correlation that is predominantly positive for the one-day differencing interval, and rather mixed for the sixteen-day interval. As Fama notes, however, it is difficult to attach much significance to these results because returns are not independent across stocks, and thus "the *sample* behavior of the market component during any given period may be expected to produce agreement among the signs of the sample serial correlation coefficients for different securities" [7, p. 74].

For the aggregate market, Fisher [10] (for U.S. data) and Dimson [6] (for British data) have reported considerable positive autocorrelation in various aggregate market indexes. Also, there is some evidence in the literature that market model regression residuals are autocorrelated.[2] For instance, Fisher [10] and Fama, Fisher, Jensen, and Roll [9] reported an average monthly serial correlation coefficient of -.12 and -.10 respectively, but they did not attach any special significance to that finding. We show in this paper that their finding was not a statistical accident and that, in fact, negative residual autocorrelation is quite pervasive. And, not only is the pattern of residual correlation clearer than for total returns, but because of the cross sectional independence of the error term, we can have greater confidence in its significance.

As noted above, it has also been observed that the market model R^2 falls systematically as the differencing interval is shortened. We present further evidence on this deterioration in R^2 below. Schwartz and Whitcomb [19] have shown that autocorrelation in returns can explain these changes in R^2. Thus, the empirical evidence is quite consistent, and it appears that the observed deterioration of R^2 is evidence of a systematic and pervasive pattern of autocorrelation in returns data.

Before turning to our evidence on autocorrelation and the deterioration of R^2, we first define the variables and sample we have used in the current study. The logarithmic form of the market model can be written as

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[1] See, for instance, Cootner [5] and Fama [8].

[2] Of course, in time series regression analysis, autocorrelation of the error term can be induced by autocorrelation of the independent variable. However, in the market model, the independent variable is simply the cross-sectional aggregation of the dependent variables, so we find it hard to believe that residual autocorrelation could be merely an artifact of index autocorrelation.

(1) $$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$

where $R_{it} = \frac{1}{N} \log_e \frac{P_t + Div_t}{P_{t-1}}$ is the log return per day on the i^{th} security, $R_{mt} = \frac{1}{N} \log_e \frac{I_t}{I_{t-1}}$ is the log of link index relatives, t is a period index, and N is differencing interval length in days.

Returns for the various differencing intervals considered are all expressed as a rate per day. The daily (log) return for any N-day differencing interval is simply the sum of the N daily returns that comprise it, divided by N:

(2) $$R^N_{i(t'=1)} = \frac{1}{N} \sum_{t=1}^{N} R_{it} .$$

From equation (2) we can write

(3) $$E(R^N_i) = E(R_i)$$

and, assuming returns independence,

(4) $$\text{Var } R^N_i = \frac{1}{N} \text{Var } R_i .$$

However, for negative autocorrelation of the R_i, variance rises *faster* with decreases in differencing interval length, while it rises slower under positive correlation.[3]

When the independence assumption underlying (4) is satisfied, the parameters of the market model equation are independent of the length of the differencing interval. It is now easy to see the effect of differencing interval length on R^2. From the standard definition of R^2, we can write

(5) $$R^2 = \frac{\beta^2_i \text{ Var } R^N_m}{\beta^2_i \text{ Var } R^N_m + \text{Var } \varepsilon^N_i} .$$

Since under independence, change in differencing interval length does not affect β and has the same proportionate effect on var R^N_m and var ε^N_i, R^2 will

[3]See Schwartz and Whitcomb [19] for a rigorous derivation of (a) the effect of autocorrelation on the time-variance relationship, and (b) the effect of differencing interval length on market model parameters and residual variance under serial independence. It is also shown that a special case pattern of nonstationarity can perturb the time-variance relationship.

not be affected by change in differencing interval length.[4] Conversely, systematic change in R^2 is evidence of serial correlation.

In the current study, we systematically varied the return measurement period (differencing interval length, N) while keeping the span of the data base constant. Using the Scholes returns tapes, we took daily, 2-day, 3-, 5-, 10-, and 20-day returns, using, in each case, data spanning the 1000-day period, June 26, 1964, to June 18, 1968. Our sample included 20 NYSE firms, listed in Table 1, which were picked at random from the S & P 500. For the market, we used a Fisher index, a simple average of all returns in each day, adjusted for cash dividends.

TABLE 1

LIST OF COMPANIES IN SAMPLE

Sample Number	Company Name	Sample Number	Company Name
1	Crown Zellerbach	11	Columbia Gas System
2	Falstaff Brewing	12	Continental Can
3	General Dynamics	13	Ford Motor
4	Gulf Oil	14	Georgia Pacific
5	International Harvester	15	National Lead
6	Mercantile Stores	16	Ohio Edison
7	Merck and Company	17	Pan Am World Airways
8	National Biscuit	18	Parke, Davis
9	Superior Oil	19	Chas. Pfizer
10	Armour	20	Stanray

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[4] While population R^2 is unaffected by a change in sample size (which in our study is brought about by changing the differencing interval length while holding constant the span of the data base), *sample* "adjusted R^2" is slightly affected due to a small bias. However, the maximum bias is only .002, and for population R^2 less than .5 (as it generally is in our study), adjusted R^2 *falls* (by at most .002) when we go from a 1-day to a 20-day differencing interval. Thus it cannot account for the sign or the magnitude of the effect we observe. See Barton [2] and Montgomery and Morrison [14]. We thank Richard Roll for bringing this point to our attention.

II. Empirical Evidence on Residual Autocorrelation and the Deterioration of R^2

We simultaneously obtained estimates of first-order serial correlation and its impact on R^2 by using, in place of ordinary least squares, the Cochorane-Orcutt iterative regression technique. This technique purges the residuals of first-order correlation (though not of higher order correlation) and gives a different (but not systematically different) estimate of beta. First consider evidence on the systematic fall of R^2. Table 2 gives 20 firm average regression statistics for ordinary least squares and for the Cochorane-Orcutt technique. Table 3 gives the firm-by-firm regression results for the daily data, and contrasts these with the twenty-day results. For compactness, the t statistics are omitted from the tables but we should note that the daily betas are not only highly significant but also have lower standard errors than those for longer differencing intervals.[5]

Turning to the regression parameters, Table 2 shows little change in average α, while average β falls throughout as we move from twenty-day to daily data. Although the firm-by-firm statistics raise some question as to whether the change in β is systematic, our findings, taken in conjunction with those of Altman, Jacquillat and Levasseur [1] and Pogue and Solnik [17] seem to indicate quite convincingly that β does fall with decreases in the differencing interval.

Pogue and Solnik suggest that the β effect is caused by omitting lagged values of the market index from the regression equation when a stock's adjustment to market change is not fully completed within a trading day. The results of our test of this hypothesis using U.S. data are reported below. Levhari and Levy [11] develop theoretically the proposition that estimated β should increase with increasing differencing intervals for stocks with true β greater than one and decrease for stocks with β less than one. The most complete empirical test of the Levhari-Levy proposition is contained in Smith [20]. Two hundred CRSP stocks were ranked by β in a prior period and divided into 10 portfolios. For all 10 portfolios average β rose monotonically as differencing interval length was increased from one through three months. β continued to increase for the high β portfolios, but for low β portfolios leveled off and then decreased (for intervals exceeding 12 months). This gives equivocal support to the Levhari-Levy hypothesis as applied to very lengthy intervals. Our own results for very short differencing intervals provide no support for the Levhari-Levy hypothesis. We should note that their analysis is in a different context than ours; they operate in return (rather than log return) space and assume intertemporal independence.

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[5] Pogue and Solnik [17] also note this, and explain it by showing that grouping daily returns into N-day intervals can cause a loss of estimator efficiency.

TABLE 2

MARKET MODEL REGRESSION RESULTS

20 COMPANY AVERAGES

Differencing Interval (Days)	Ordinary Least Squares			Cochorane - Orcutt				
	α	β	$\bar{R}^2$	α	β	ρ	No. of $\rho > 0$	$\bar{R}^2$
1	-.00027	.879	.136	-.00031	.878	+.022	15	.141
2	-.00031	.893	.163	-.00031	.892	-.011	8	.168
3	-.00033	.901	.183	-.00033	.904	-.040	8	.195
5	-.00026	.902	.190	-.00027	.899	-.064	3	.200
10	-.00037	.911	.252	-.00033	.918	-.073	6	.262
20	-.00026	.914	.323	-.00037	.889	-.111	5	.342

TABLE 3

MARKET MODEL REGRESSION RESULTS: ONE-DAY DIFFERENCING INTERVAL

20 COMPANIES

Ordinary Least Squares (OLSQ) and Cochorane - Orcutt (CORC) Regressions

		One-Day Regressions					Comparison with 20-Day Regressions			
Firm		α	β	ρ	$\bar{R}^2$	VAR E	$\frac{\alpha_1-\alpha_{20}}{\lvert\alpha_{20}\rvert}$	$\frac{\beta_1-\beta_{20}}{\beta_{20}}$	$\frac{\bar{R}^2_1-\bar{R}^2_{20}}{\bar{R}^2_{20}}$	$\frac{\text{VAR } E_1}{\text{VAR } E_{20}}$
1	OLSQ	-.00058	.657		.106	.0001304	+.318	-.344	-.720	19.46
	CORC	-.00056	.652	+.040	.108	.0001302	+.282	-.310	-.725	19.43
2	OLSQ	-.00043	.401		.019	.0003031	+.494	-.552	-.932	36.66
	CORC	-.00043	.394	+.033	.020	.0003031	+.481	-.558	-.929	35.24
3	OLSQ	-.00041	1.490		.191	.0003381	-3.500	+.362	-.112	18.99
	CORC	-.00041	1.495	+.066	.195	.0003432	-1.615	+.392	-.141	19.39
4	OLSQ	-.00020	1.231		.242	.0001713	+.622	-.228	-.645	29.53
	CORC	-.00019	1.223	+.047	.244	.0001711	+.677	-.231	-.642	29.50
5	OLSQ	-.00006	.650		.161	.0000801	-1.300	+1.070	+1.176	15.70
	CORC	-.00005	.660	+.110	.171	.0000789	-1.192	+2.143	+.125	17.15
6	OLSQ	-.00067	.837		.158	.0001357	+.118	-.051	-.597	27.69
	CORC	-.00066	.827	+.065	.162	.0001353	+.096	-.053	-.598	27.61
7	OLSQ	+.00061	.328		.031	.0001245	+.017	+.035	-.295	13.83
	CORC	+.00064	.310	+.121	.045	.0001227	+.143	+.020	+.071	13.48
8	OLSQ	+.00044	.707		.101	.0001631	+.375	-.089	-.612	23.30
	CORC	+.00044	.704	+.020	.102	.0001632	+.043	+.220	-.707	26.32
9	OLSQ	-.00041	.484		.057	.0001433	.079	+.244	-.472	28.10
	CORC	-.00041	.484	-.021	.057	.0001434	.025	+.186	-.610	28.68
10	OLSQ	-.00086	1.171		.169	.0002396	+.122	-.125	-.645	29.58
	CORC	-.00087	1.168	+.050	.172	.0002391	+.121	-.176	-.652	29.89

TABLE 3 (Cont'd)

Firm		α	β	ρ	$\bar{R}^2$	VAR E	$\frac{\alpha_1-\alpha_{20}}{\lvert\alpha_{20}\rvert}$	$\frac{\beta_1-\beta_{20}}{\beta_{20}}$	$\frac{\bar{R}_1^2-\bar{R}_{20}^2}{\bar{R}_{20}^2}$	$\frac{\text{VAR } E_1}{\text{VAR } E_{20}}$
11	OLSQ	-.00011	.289		.036	.0000796	+.353	-.190	-.774	28.43
	CORC	-.00012	.299	-.175	.066	.0000773	+.294	-.165	-.614	27.60
12	OLSQ	+.00008	.700		.113	.0001373	+1.615	-.254	-.725	26.40
	CORC	+.00009	.695	+.026	.114	.0001374	+2,286	-.205	-.740	27.48
13	OLSQ	-.00054	.940		.228	.0001079	-.149	+.111	-.421	23.98
	CORC	+.00056	.954	+.108	.237	.0001067	-.191	+.156	-.406	23.71
14	OLSQ	-.00018	1.228		.242	.0001713	+.660	-.231	-.622	29.53
	CORC	-.00017	1.219	+.046	.244	.0001711	+.702	-.234	-.619	29.50
15	OLSQ	-.00043	.594		.139	.0000796	+.246	-.184	-.636	22.74
	CORC	-.00042	.595	+.006	.139	.0000796	+.250	-.144	-.658	23.41
16	OLSQ	-.00001	.318		.029	.0001228	+.909	-.236	-.828	35.09
	CORC	-.00001	.317	-.092	.037	.0001219	+.929	-.191	-.802	35.85
17	OLSQ	-.00120	2.042		.288	.0003785	-1.069	+.855	+.286	22.01
	CORC	-.00123	2.070	+.042	.289	.0003771	-1.236	+.842	+.246	20.38
18	OLSQ	-.00078	1.304		.170	.0003070	-.026	+.160	-.547	35.70
	CORC	-.00078	1.304	-.0002	.170	.0003073	-.040	+.155	-.550	35.32
19	OLSQ	+.00006	.725		.104	.0001685	+1.231	-.279	-.685	20.30
	CORC	+.00006	.714	+.054	.107	.0001681	+1.194	-.277	-.674	20.25
20	OLSQ	-.00037	1.484		.134	.0005346	+.422	-.057	-.741	56.87
	CORC	-.00037	1.473	-.110	.144	.0005285	+.393	-.021	-.725	56.22
				No. > 0	OLSQ		14	7	2	
					CORC		14	8	3	
		Bernoulli Probability			OLSQ		.058	.132	.000	
					CORC		.058	.252	.001	

We next consider the autocorrelation structure in the market model residuals. The Cochorane-Orcutt regressions suggest a decided pattern of first-order serial correlation. The daily data show positive correlation, while all other intervals give negative ρ, and ρ becomes more strongly negative through the 20-day differencing interval.[6] While the magnitude of ρ in each individual regression is generally rather small, the pattern appears to be quite consistent across firms, as is seen in Table 4 which reports the full pattern by firm and by differencing interval. For instance, 15 out of 20 daily ρ are positive. Given independence between the ρ_i, by Bernoulli trial, the probability of this occurrence is .02 if positive and negative ρ are equally likely. As we pass through the 2- and 3-day differencing intervals, the pattern turns negative, and becomes overwhelmingly negative for the 5- to 20-day intervals.

When we look at the serial correlation of the market index for our 1000-day period, the reason for the observed drop in R^2 becomes quite evident. The first-order correlation of the market index is positive for every differencing interval: .29 for 1-day, .15 for 2-days, .18 for 3, .30 for 5, .21 for 10 and .09 for 20-days.[7] No wonder then that R^2 drops each time we shorten the differencing interval--nearly every firm has residual correlation that is arithmetically smaller than market index correlation for every differencing interval.

Unfortunately, however, while the Cochorane-Orcutt regressions help to uncover a correlation pattern, they improve the market model fit only slightly. As seen from Table 2, adjusted R^2 rises a little on average and the α and β coefficients are not systematically affected (and they should not be), but the deterioration of R^2 as the differencing interval is shortened is still pronounced. We believe there are two main reasons for this: First, we have removed first-order autocorrelation from the residuals, but the positive market index correlation remains and continues to cause R^2 to fall. Second, considerable negative higher order correlation in the daily residuals is strongly suggested by the increasingly negative first-order correlation as the differencing interval increases; and as noted, the Cochorane-Orcutt technique does not eliminate this correlation.

In conclusion, our evidence on residual autocorrelation quite strongly suggests that daily residuals are positively correlated, while the first-order correlation for longer period data is predominantly negative. The evidence on

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[6] For the 20-day differencing interval, our 20-firm average ρ(-.111) is consistent with the monthly estimates, -.12 and -.10 reported, respectively, by Fisher [10] and by Fama, Fisher, Jensen and Roll [9].

[7] Fisher [10, p. 206] reports a + .19 first-order serial correlation coefficient for the monthly Combination Price Index, for the period 1926-60.

TABLE 4

FIRST-ORDER CORRELATION OF MARKET MODEL RESIDUALS

20 FIRMS, 6 DIFFERENCING INTERVALS

	Differencing Interval (Days)					
Firm	1	2	3	5	10	20
1	+.040	-.010	-.032	-.027	-.030	-.146
2	+.033	-.102	-.259	-.049	-.116	+.023
3	+.066	+.016	+.039	+.009	+.028	-.138
4	+.047	-.026	-.081	-.115	-.091	+.059
5	+.110	+.066	+.009	-.042	+.004	-.305
6	+.065	+.031	+.024	-.116	-.139	-.119
7	+.121	+.224	+.221	+.212	+.015	+.035
8	+.020	+.055	-.025	-.033	-.083	-.372
9	-.021	+.019	-.000	-.046	-.051	-.208
10	+.050	-.031	-.124	-.167	-.179	-.132
11	-.175	-.117	+1069	+.053	-.208	-.121
12	+.026	+.064	-.053	-.080	-.131	-.226
13	+.108	-.057	-.117	-.057	+.032	-.114
14	+.046	-.026	-.080	-.115	-.092	+.059
15	+.006	-.014	+.048	-.111	-.287	-.210
16	-.092	-.090	-.108	-.063	-.002	-.164
17	+.042	-.018	+.010	-.029	+.110	+.080
18	-.000	-.140	-.188	-.232	+.060	-.065
19	+.054	+.010	+.019	-.033	-.151	-.018
20	-.110	-.073	-.179	-.231	-.146	-.132
Avg.	+.022	-.011	-.040	-.064	-.073	-.111
Avg. of Abs. Val.	.062	.059	.084	.091	.097	.136
No > 0	15	8	8	3	6	5
Bernoulli Prob.	.021	.252	.252	.001	.058	.021

residual autocorrelation, taken in conjunction with the observed positive market index returns autocorrelation clearly explains the deterioration of R^2. We now consider a variety of factors which might explain the observed correlation patterns.

III. The Causes of Residual Autocorrelation

Measurement Error

The first thing one might think of in explaining the deterioration of R^2 is the possibility that measurement error affects daily data more than longer period data. For example, recording and keypunching errors will increase unexplained variance and therefore reduce R^2. If the errors are independent from observation to observation, then by the pure time-variance relationship, the unexplained variance introduced by measurement errors rises with decreases in the differencing interval at the same rate as total variance and market index variance, so by equation (5) R^2 is not affected.

However, an error in one price generates two adjacent daily returns errors that are necessarily of opposite sign. Thus a returns series with this kind of error will show some negative autocorrelation. Then, by the time-variance relationship under dependence, one-day returns variance will be more than twice two-day returns variance, while market index variance will just double and R^2 will fall as the differencing interval is reduced. We tried to reduce this kind of error by flagging and verifying large returns. The evidence we presented above about *positive* one-day correlation strongly suggests that most of the observed change in R^2 is not accounted for by this type of measurement error.

The 1/8 effect

Another possible cause of autocorrelation in the residuals that is in many ways analogous to measurement error is what we call the 1/8 effect. New York Stock Exchange prices are quoted in minimum units of 1/8 (except for prices under \$10, where 16ths are sometimes used). Thus, what would be a smooth price series is in effect rounded and becomes a lumpy series. While it is clear that this increases the variance of returns,[8] it is not obvious that it will distort the pure time-variance relationship and cause R^2 to change systematically as we change the differencing interval. However, rounding to the nearest eighth introduces some negative correlation[9] and thus causes the variance of lumpy returns

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[8] The reason for increasing variance is that while a round *away* from the mean is as likely as a round *toward* the mean, the squared deviations of rounds away from the mean will outweigh the squared deviations of rounds toward the mean.

[9] A positive return containing a round *up* is more likely than a positive return containing a round *down* (because the latter implies a larger unrounded

to rise faster than the variance of smooth returns as the differencing interval is shortened.

To assess the magnitude of the 1/8th effect on variance, we ran a simulation which is reported in Table 5. We chose a range of prices and standard deviations of returns that just about bracketed the stocks in our sample. For each price and standard deviation, 20 series of smooth prices were generated from a normal returns distribution (with mean of 0) and then "lumpified." The numbers in Table 5 are the 20-iteration averages of lumpy divided by smooth sample variance for each differencing interval.

Clearly, the lowest priced stocks and the least volatile stocks have the greatest 1/8 effect.[10] The important property for our purposes, however, is that, for any given price and standard deviation, the variance of lumpy returns relative to smooth returns rises as the differencing interval is shortened from 50 days to 1 day. This raises the possibility that the 1/8 effect could, in part, explain our regression results.

This does not seem to be the case, however. Few standard deviations in our sample were much below .01 and none was anywhere near .002, and prices were generally above $10. When we simulated the impact of the 1/8 effect on the regression parameters and R^2 using a standard deviation of .01 and a price of $15, we found only a 5 percent reduction in the daily market model R^2. While it is possible for the 1/8 effect to have a considerable impact on market model regressions for very low priced stocks, we do not believe it had much impact on the regressions we ran in this study.

Excluded variables

Clearly, omission of an explanatory variable can lower R^2; also, if the omitted variable is correlated with the market, it can bias the estimated beta parameter, and, further, if it is *auto*correlated, it can account for autocorrelated residuals. Whether R^2 is reduced more for short differencing intervals depends on what happens to the variance of the omitted variable and to its

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return). If P_t contains a round up, then, in the next period, the probability of falling back to the next lower fractional price is greater than the probability of reaching the next higher fractional price. Similarly, if the return contains a round down, the next return is more apt to be positive than negative. Thus, a positive return in t is more likely to be followed by a negative than by a positive return in t+1. The treatment of negative returns is symmetric.

[10] The lower the price, the greater the return implied by a price change of 1/8; the lower the standard deviation (given a mean of 0), the less likely is a 1/8 return to occur in the absence of rounding; thus for low price, low standard deviation stocks, rounding is more apt to generate abnormally large returns, thereby inflating variance.

TABLE 5

SIMULATION OF "1/8 EFFECT"

RATIO OF "LUMPY" TO "SMOOTH" VARIANCES

		Differencing Interval ("Days")					
		1	2	5	10	20	50
S.D.	Price						
.002	2	12.16	8.30	5.12	3.68	2.66	1.56
	10	4.02	2.73	1.81	1.44	1.20	1.07
	50	1.26	1.21	1.03	1.00	.96	.95
	100	1.07	1.02	1.00	.99	.97	.95
.01	2	2.47	1.77	1.29	1.14	1.04	.98
	10	1.15	1.07	1.01	.99	.96	.96
	50	1.01	1.00	.99	.98	.95	.96
	100	1.00	1.00	.99	.98	.95	.96
.05	2	1.27	1.13	1.04	1.00	.97	.96
	10	1.01	1.00	.99	.98	.95	.95
	50	1.00	.99	.98	.98	.95	.96
	100	1.00	.99	.98	.98	.95	.96

correlation with the market as the differencing interval gets shorter.

The excluded variable we considered is the lagged market. The lagged market might have some explanatory power if the adjustment of stock prices to aggregate market movements is not instantaneous. In fact, Fisher [10] has stated, on the basis of indirect evidence, that there may be substantial lags in the adjustment of many stock prices to market index changes. In addition, Fisher, and Pogue and Solnik [17] note several institutional-technical factors that could cause a very short lag (one day or less).[11] Most prominent of these is the "Fisher effect"; when some stocks trade rather infrequently, their closing transaction prices lag behind those of more frequently traded stocks, introducing spurious positive correlation in the market index because market returns are averages of temporally ordered data.[12]

To test the lagged market relationship, we ran regressions of the form

$$(6)\quad R_{it} = \alpha_i + \beta_{it}R_{mt} + \beta_{i(t-1)}R_{m(t-1)} + \beta_{i(t-2)}R_{m(t-2)} + \beta_{i(t-3)}R_{m(t-3)} + \varepsilon_{it} .$$

The results are reported in Table 6. For the 20-day differencing interval, we discern no meaningful lag pattern. For the 1-day regressions, only the 1-day lag showed enough significant β_{t-1} to suggest a meaningful lag pattern (8 of 20 β_{t-1} were significant at the .05 level, and 4 at the .01 level). The β_{t-1} appear to be distributed around zero.

Despite the weak evidence that a short market lag exists for a number of stocks, inclusion of the lagged market does not alter the principal results reported in Tables 2 and 3: (1) As the differencing interval is shortened, we still observe a large number of β_t falling (14 of 20), so the bias in β_t appears to persist. (2) R^2 remains substantially less for the 1-day than for the 20-day differencing interval. (3) The first-order autocorrelation pattern remains practically identical--ρ is predominantly positive for the 1-day differencing interval, and negative for the 20-day differencing interval.

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[11] Pogue and Solnik note that on some European exchanges the index is calculated on prices registered at some time before the market close, thus causing the index to lag returns computed from closing prices. This problem clearly does not exist in the current study, since we use daily closing prices in computing the Fisher index.

[12] See Working [21] for a demonstration that a time series of averages obtained from temporally ordered data will tend to be positively autocorrelated.

TABLE 6

LAGGED MARKET MODEL REGRESSION RESULTS:†

ONE- AND TWENTY-DAY DIFFERENCING INTERVALS

20 Companies

	ONE-DAY REGRESSIONS					TWENTY-DAY REGRESSIONS				
Firm	β_t	β_{t-1}	β_{t-2}	$\hat{\rho}$	$\bar{R}^2$	β_t	β_{t-1}	β_{t-2}	$\hat{\rho}$	$\bar{R}^2$
1	+.620**	+.128*	+.094	+.040	.113	+1.043**	-.198	+.109	-.022	.417
	9.80	1.94	1.42			5.58	1.06	.58		
2	+.339**	+.189*	-.050	+.034	.026	+.892**	+.255	-.245	+.052	.326
	3.52	1.87	.49			4.26	1.21	1.16		
3	+1.525**	-.076	+.004	+.064	.193	+1.068**	-.359	+.007	-.050	.282
	14.94	.71	.04			3.56	1.19	.22		
4	+1.208**	+.104	-.051	+.050	.244	+1.605**	+.035	-.080	+.047	.645
	16.61	1.37	.67			8.99	.20	.45		
5	+.652**	+.009	-.039	+.115	.160	+.326*	-.207	-.207	-.306	.146
	13.09	.17	.76			2.03	1.29	1.28		
6	+.840**	+.063	-.006	+.060	.166	+.881**	-.124	+.238	-.104	.426
	13.02	.94	.09			5.50	.77	1.48		
7	+.267**	+.201**	+.119*	+.115	.049	+.319	+.214	+.315	+.086	.134
	4.34	3.13	1.85			1.50	1.01	1.47		
8	+.698**	+.075	-.032	+.023	.103	+.836**	-.552**	-.067	-.359	.399
	9.84	1.01	.43			4.71	3.10	.37		
9	+.516**	-.056	-.019	-.022	.061	+.409**	+.177	-.220	-.240	.231
	7.76	.78	.27			2.63	1.13	1.41		
10	+1.218**	-.198*	+.058	+.052	.178	+1.376**	+.261	.075	-.043	.574
	14.22	2.21	.65			7.22	1.36	.39		
11	+.268**	+.066	-.012	-.172	.038	+.368**	+.081	-.118	-.108	.212
	5.40	1.28	.22			3.08	.67	.98		
12	+.658**	+.141*	+.006	+.025	.118	+.957**	-.082	-.473	-.194	.421
	10.12	2.07	.09			5.72	.47	.28		
13	+1.003**	-.170**	+.054	+.109	.239	+.841**	-.219	+.081	-.076	.444
	17.48	2.84	.91			5.57	1.44	.53		
14	+1.208**	+.104	-.051	+.050	.244	+1.605**	+.351	-.080	+.047	.645
	16.61	1.37	.67			8.99	.20	.45		
15	+.582**	+.077	+.091*	+.006	.148	+.751**	-.273*	-.021	-.229	.436
	11.79	1.50	1.77			5.66	2.05	.16		
16	+.331**	-.025	+.007	-.087	.030	+.429**	-.012	-.082	-.162	.190
	5.37	.39	.11			3.14	.09	.60		
17	+2.354**	-.936**	+.130	+.048	.343	+1.115**	+.215	+.307	+.130	.275
	22.64	8.62	1.19			3.75	.72	1.02		
18	+1.434**	-.323**	.171*	+.009	.184	+1.131**	-.122	-.106	-.042	.381
	14.84	3.20	1.69			5.22	.56	.49		
19	+.719**	+.072	-.028	+.056	.105	+1.039**	-.290	-.484**	-.095	.440
	9.97	.96	.37			5.28	1.47	2.44		

TABLE 6 (Cont'd)

Firm	ONE-DAY REGRESSIONS β_t	β_{t-1}	β_{t-2}	$\hat{\rho}$	$\bar{R}^2$	TWENTY-DAY REGRESSIONS β_t	β_{t-1}	β_{t-2}	$\hat{\rho}$	$\bar{R}^2$
20	+1.506**	+.058	-.102	-.107	.137	+1.577**	-.233	-.118	-.124	.536
	11.74	.43	.76			7.06	1.04	.53		
No.>0	20	13	9	16		20	8	6	5	
Bernoulli Prob.	.000	.132	.412	.006		.000	.252	.058	.021	

† The estimating equation also included $R_{m(t-3)}$; betas for this variable are excluded from the table for compactness. Immediately below the betas are their t statistics. Significance for d.f. = 995 is 1.64 at the .05 level(*), and 2.33 at .01 (**), and for d.f. = 45 is 1.68 at .05, and 2.41 at .01. ρ is estimated from the Durbin - Watson Statistic, with $\hat{\rho} = (2-DW)/2$.

The Impact of Market Makers

Market makers can be taken to be NYSE specialists and any trader who places limit orders with the specialists. As discussed in Schwartz and Whitcomb [18], the operation of specialists, as well as the limit orders of other traders, can establish reflecting barriers which introduce negative autocorrelation into short period returns. However, it is also possible that attempts of stock exchange specialists to make an orderly market by preventing large transaction-to-transaction price changes generate some positive autocorrelation in transaction-to-transaction returns. While the net impact of the specialist is not theoretically determinate, it is possible that the observed positive correlation in daily residuals is a reflection of the impact of the specialist. Also, it is possible that other large traders (such as the institutions) introduce positive autocorrelation into the data by "breaking up" their large trades into a contiguous series of smaller transactions. Direct evidence of a specialist impact (mainly on thinly traded issues) is given by Cohen, Maier, Ness, Okuda, Schwartz, and Whitcomb [3]; indirect evidence that specialist intervention introduces positive autocorrelation in daily data is provided by Yamada [22] who found that, for stocks listed on the first section of the Tokyo exchange where there is no specialist, daily returns are predominantly negatively autocorrelated. On the other hand, Cohen, Maier, Ness, Okuda, Schwartz, and Whitcomb [3] find that institutional trading has but a small impact on NYSE returns data, as evidenced by an only weakly significant elasticity of returns variance with respect to institutional holdings.

The Fisher Effect and Portfolio Rebalancing

Negative residual and positive market index autocorrelation can be caused simultaneously if prices of individual stocks do not move concurrently with the aggregate market. For instance, if some stocks (group A) experience price movements by the close of trading on day j, while for other stocks (group B) the movement is delayed until day j + n, the aggregate index will tend to adjust in the same direction on both j and j + n, and group A (B) stocks will have positive (negative) market model residuals on day j and negative (positive) residuals on day j + n. Hence, aggregate market returns will be positively autocorrelated, while market model residuals will be negatively autocorrelated for both the A and B group stocks.

Essentially, this is the mechanism used by Lawrence Fisher [10] to explain what has come to be called the "Fisher effect." Fisher attributed this effect partly to the fact that infrequently traded issues often "close" (i.e., have their last recorded price) well before the end of the trading day.[13] Cohen,

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[13]Because the importance of the Fisher effect is expected to be related to the thinness of the market for a company's stock, Dimson [6] suggested that

Maier, Schwartz, and Whitcomb [4] present a rigorous model of the Fisher effect which shows that lags in the adjustment of individual stock transaction prices following a demand shift can be related to random tender arrival rates.[14] This model suggests that:

(1) the Fisher effect need not imply mechanical-technical delays in transmitting and executing trading orders; (2) the price adjustment lags can be one day or longer, but for an issue the probability of an n-day lag falls quickly as n rises; (3) across issues, the probability of an n-day lag falls directly as the value of shares outstanding rises. Thus the random tenders arrival model of the Fisher effect would lead us to expect fairly small residual autocorrelation in the widely-held S & P 500 stocks comprising our sample (but somewhat more substantial correlation in the market (Fisher) index, and thus deterioration of R^2) and rapidly diminishing residual and index autocorrelation as the differencing interval is lengthened.

However, the persistently positive autocorrelation of value-weighted indexes, even for differencing intervals as long as a month, which both we and Fisher have observed, suggests that the Fisher effect encompasses more than lagged price changes due to thinness: "I suspect that there are stocks whose equilibrium prices are affected by changes in the level of the general market index but with a substantial delay--considerably longer than the maximum 'excusable' delay of a trading day implied by the method of collecting data" [10, p. 199]. This impression is reinforced by our results which suggest a pervasive pattern of negative higher order correlation in daily residuals.

Our hypothesis on the "longer period" Fisher effect and the associated negative residual autocorrelation for longer differencing intervals is somewhat more tentative. It may be caused by delayed attempts on the part of investors to rebalance their portfolios after a price change in some stocks has distorted their portfolio value weights.[15] That is, let some news occur on day j that has

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broadly based, equally weighted indexes should be more strongly positively autocorrelated than value or price weighted indexes (which reflect primarily the movements of the more resilient issues). Dimson's empirical findings, based on British data, appear to confirm his hypothesis, as do Moore's [15] and Fisher's previous findings with American data.

[14] The spirit of the Cohen, Maier, Schwartz, Whitcomb model is that, while new information for instance can cause a shift in the market demand function for a stock and so alter bid and ask prices, a transaction (and thus a transaction price) can occur only if individual investors change their ordering along the market demand function. In other words, a monotonic revision of individual demands will not trigger a transaction, while a random tender will. Thus transactions must await the arrival of random tenders, so transaction prices can be less contemporaneous than bid-ask prices.

[15] See Merton [13] for a model of continuous portfolio rebalancing.

its primary impact on only some stocks--those in group A, for instance. On day j, these will be the "active stocks" that cause the market movement. However, the price changes for the A stocks will (with price of B stocks constant) cause portfolio weights to stray from their desired values; thus, as investors seek to adjust the weights, the price of the "passive" stocks will be affected on, say, day j + n. This then results, as noted above, in the positive index and negative residual autocorrelation.

Given a distribution (across individuals) of transaction costs of portfolio rebalancing, the price adjustment for any "passive" issue will occur over a number of days, with the size of the daily adjustment not necessarily decaying rapidly as n increases. Also, portfolio rebalancing occurs in widely-held issues as well as thin issues. Therefore it is possible that the longer period autocorrelation pattern we have observed is explained largely by delayed portfolio rebalancing.

The Fisher effect hypothesis (which can encompass delayed portfolio rebalancing) is similar to the lagged market hypothesis discussed earlier, but differs in an important respect: if the set of stocks that "move first" is not constant over time, but rather depends on the specific "news" that causes price to change, then the negative residual correlation pattern can be stable but the lagged market will not have a consistent relationship with current stock returns.[16] Thus the lagged market hypothesis might be viewed as a special case of the portfolio balancing hypothesis. We have observed, except for the 1-day lag (where the evidence is weak), no meaningful evidence of a lagged market relationship. Negative autocorrelation, on the other hand, appears to be general across stocks and to persist up to rather lengthy differencing intervals.[17]

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[16]The relationship with $R_{m(t-1)}$ will be negative for stocks that consistently "move first," positive for those that consistently lag, and will tend toward zero for those that switch back and forth between categories.

[17]Note that delayed portfolio rebalancing can lead to trading profits in excess of commission costs and a required rate of return only (1) if traders can identify a negative residual as being caused by the stock lagging an aggregate market movement rather than by its having led some previous market movement or by some firm specific event; or, (2) if some securities consistently experience delayed portfolio rebalancing more than do others (our insignificant lagged market results provide no support of this possibility). Thus we have no *a priori* reason to expect delayed portfolio rebalancing to be inconsistent with efficient capital markets.

IV. Conclusion

In this paper we have shown that residual and market index returns autocorrelation explain the observed deterioration of R^2 as the market model regression equation is estimated from shorter period data. In so doing, we have presented evidence of a systematic pattern of residual autocorrelation across the stocks in our sample and have considered various factors which might cause such correlation.

Knowledge that residual and market index returns autocorrelation is the cause of the low R^2 observed for short period data suggests that, through the use of appropriate econometric techniques, reasonably good market model estimation may be obtained from weekly and perhaps even daily data. The ability to use such data would facilitate the analysis of issues such as the stationarity of beta. And, given some degree of nonstationarity, it would enable investors to compute purer, more current estimates of beta.

Our investigation of various causes of autocorrelation has considered both technical and economic factors (although we do not wish to imply that our list is necessarily exhaustive). We have reached the following conclusions:

1. Technical factors such as data errors and the 1/8 effect explain little of the correlation patterns we have observed.
2. The impact of market makers and institutional investors explains the generally positive residual autocorrelation for the one-day differencing interval.
3. There is considerable evidence in favor of a short-term Fisher effect. The observed positive index autocorrelation could well result from the short-term Fisher effect impacting on thin issues (which are reflected in the Fisher index); the presence of the Fisher effect is also indicated by the generally negative residual autocorrelation observed for differencing intervals of two days and longer.
4. Evidence supportive of the delayed portfolio rebalancing hypothesis is given by the persistence of positive index autocorrelation in longer differencing interval data (up to at least one month) and by the presence of negative higher-order residual correlation in daily data (as evidenced by the strength of negative first-order correlation for longer differencing intervals).

Further investigation of the causes of residual autocorrelation would clearly be desirable. For instance, a cross-sectional analysis of the relationship between market thinness and first-order correlation in daily residual returns, and further comparison of correlation patterns in broadly based versus value-weighted market indexes along the lines started by Dimson [6], might enable us to derive a more definitive conclusion about the importance of shorter-run

Fisher effects. Also, additional empirical investigation should be undertaken to test directly the portfolio rebalancing hypothesis.

At this stage, it appears that economic factors such as the impact of market makers and delayed portfolio rebalancing seem to provide the strongest explanations of the correlation patterns we have observed. However, the portfolio rebalancing hypothesis is especially conjectural and in need of further investigation.

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