Nonsynchronous Security Trading and Market Index Autocorrelation

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ABSTRACT

The theoretical portfolio autocorrelation due solely to nonsynchronous trading is esti-
mated from a derived model. This estimated level is found to be substantially less than
that observed empirically. The theoretical and empirical relationship between portfolio
size and autocorrelation also is investigated. The results of this study suggest that other
price-adjustment delay factors in addition to nonsynchronous trading cause the high
autocorrelations present in daily returns on stock index portfolios.

THIS PAPER INVESTIGATES THE extent to which nonsynchronous trading explains
observed autocorrelations in daily returns on stock market indices. Market-index
autocorrelation by itself is of limited interest. However, knowledge concerning
the source of price-adjustment delays causing this autocorrelation is very signifi-
cant for a better understanding of the price-formation process. While observed
daily returns on individual stocks exhibit, on average, only slightly positive first-
order autocorrelations, market indices exhibit pronounced positive values. The
source of this strong index autocorrelation and the extent to which its magnitude
is explained by a particular nonsynchronous trading model are examined here.

An estimate of the implied theoretical portfolio autocorrelation for portfolios
of different sizes is derived from the Scholes and Williams [9] model of nonsyn-
chronous trading. Parameters in this model are estimated based upon a random
sample of 280 NYSE firms with known trading frequencies over a period of time.
The derived model explains well the empirical autocorrelation pattern as firms
are added to the portfolio. However, the level of autocorrelation observed greatly
exceeds that predicted. The implication is that other price-adjustment delay
factors, in addition to nonsynchronous trading, play a major role in determining
market-return autocorrelation.

Sections I and II review nonsynchronous trading effects and develop the model
of portfolio autocorrelation. Section III describes the data sample employed and
presents the theoretically derived estimates of the equal- and value-weighted
market-index autocorrelations. Section IV contains the major empirical results.
The last section presents the conclusions and implications.

I. Nonsynchronous Trading Effects

Fama [5] found slightly positive average autocorrelations in examining daily
security returns with a lag of one day and no empirical evidence of significant
autocorrelations for higher lags. Yet daily market-index returns exhibit a pro-

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nounced positive first-order autocorrelation. This index phenomenon has been called the Fisher effect since Lawrence Fisher [6] hypothesized its probable cause. Hawawini [7] found positive first-order cross-correlations between security returns and diminishing positive cross-correlations for larger lags. One possible explanation for all these empirical departures from the efficient-markets model is nonsynchronous trading.

Observed security price changes occur at different times throughout the trading day. Reported daily returns only reflect the last trade that took place. Thus, there is often a substantial divergence between reported transaction-based returns and true returns, especially for less active issues. Any use of reported daily returns as a proxy for true returns results in the econometric problem of errors in variables.

Scholes and Williams [9, 10] and Cohen, Maier, Schwartz, and Whitcomb [3, 4] show how this nonsynchronous security trading will induce spurious auto- and cross-correlations into individual-security and market-index returns. Several implications of their theoretical models are important to this paper. First, individual-security daily returns based on observed transaction prices should be slightly negatively first-order autocorrelated. Second, first-order cross-correlations between securities will not equal zero and should be predominantly positive. Third, market-index returns based on transaction prices should be positively first-order autocorrelated, and this induced positive autocorrelation will be more severe when more weight is given to thinly traded securities, as in an equal-weighted market index.

Cohen, Maier, Schwartz, and Whitcomb [3, 4] and Cohen et al. [2] place nonsynchronous trading in a broader class of market frictions, which may induce price-adjustment delays into the trading process. The nonsynchronous trading effect is, then, only one of several factors that may contribute to the presence of autocorrelation in market-index returns.

The Scholes and Williams and Cohen et al. studies have determined the possible algebraic sign of the market-index autocorrelation arising from nonsynchronous trading. This study will apply the Scholes and Williams model to determine the magnitude of the market-index autocorrelation. This implied market-index autocorrelation due to nonsynchronous trading will then be compared with the observed autocorrelation to determine the amount attributable to nonsynchronous trading and, consequently, the amount attributable to other price-adjustment delay factors.

II. Portfolio-Return Autocorrelation

The first-order autocorrelation of the transaction-based return on a portfolio is

$$\text{Corr}(\tilde{R}_{pt}^T, \tilde{R}_{pt-1}^T) = \text{Cov}(\tilde{R}_{pt}^T, \tilde{R}_{pt-1}^T)/\text{Var}(\tilde{R}_{pt}^T), \tag{1}$$

where

$$\tilde{R}_{pt}^T = \sum_{i=1}^n x_i \tilde{R}_{it}^T, \qquad \text{subject to } \sum_{i=1}^n x_i = 1, \tag{2}$$

and the superscript T emphasizes that these are observed or transaction-based

returns rather than true returns. Substituting (2) into (1) leads to

$$\text{Corr}(\tilde{R}_{pt}^{T}, \tilde{R}_{pt-1}^{T})$$

$$= \frac{\sum_{i=1}^{n} x_i^2 \text{Cov}(\tilde{R}_{it}^{T}, \tilde{R}_{it-1}^{T}) + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} x_i x_j \text{Cov}(\tilde{R}_{it}^{T}, \tilde{R}_{jt-1}^{T})}{\sum_{i=1}^{n} x_i^2 \text{Var}(\tilde{R}_{it}^{T}) + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} x_i x_j \text{Cov}(\tilde{R}_{it}^{T}, \tilde{R}_{jt}^{T})} . \tag{3}$$

If the portfolio weights are equal $(x_i = 1/n)$, then (3) reduces to

$$\text{Corr}(\tilde{R}_{pt}^{T}, \tilde{R}_{pt-1}^{T})$$

$$= \frac{(1/n)[\text{average autocovariance}]^{T} + [(n-1)/n][\text{average cross-covariance}]^{T}}{(1/n)[\text{average variance}]^{T} + [(n-1)/n][\text{average covariance}]^{T}} . \tag{4}$$

Equation (4) specifies the transaction-based analytical relationship between portfolio autocorrelation and individual-firm and pairwise return parameters. Perry [8] alludes to the theoretical portfolio autocorrelation for large stocks in the absence of nonsynchronous trading as being a variance-weighted average of individual-firm return autocorrelations. This conceptualization is incorrect when security returns contemporaneously covary, as is well documented empirically. Examining equation (4), the portfolio autocorrelation will reduce to the weighted average referred to in Perry only if the average cross-covariance term and the average contemporaneous covariance term are both zero. Below we introduce in (3) the relationship between true return parameters and the transaction-based return parameters.

Scholes and Williams [9] assumed that transactions for individual securities arise following independent Poisson processes with transaction arrival rates λ_i.[1] They established the relationship between the transaction-based terms in equation (3) and the variance-covariance matrix of the underlying return process of the unobservable true returns (see the Appendix).

From the market model in the true returns, we know that

$$\text{Cov}(\tilde{R}_{it}, \tilde{R}_{jt}) = \beta_i \beta_j \text{Var}(\tilde{R}_{mt}). \tag{5}$$

Introducing (5) into (3) along with the relationships between the true return parameters and the transaction-based return parameters according to the Scholes and Williams [9] model, the observed transaction-based value-weighted portfolio autocorrelation is

$$\text{Corr}(\tilde{R}_{pt}^{T}, \tilde{R}_{pt-1}^{T})$$

$$= \frac{\sum_{i=1}^{n} x_i^2 [\delta_{\lambda_i v_i} \text{Var}(\tilde{R}_{it})] + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} x_i x_j [\alpha_{\lambda_i \lambda_j} \beta_i \beta_j] \text{Var}(\tilde{R}_{mt})}{\sum_{i=1}^{n} x_i^2 [\theta_{\lambda_i v_i} \text{Var}(\tilde{R}_{it})] + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} x_i x_j [\gamma_{\gamma_i \lambda_j} \beta_i \beta_j] \text{Var}(\tilde{R}_{mt})} , \tag{6}$$

[1] The assumption of Poisson-distributed transaction arrivals may be challenged on the basis that trades occur more frequently at the beginning and close of the trading day. This reality should lessen the nonsynchronous bias effect present when estimating parameters calculated using transaction data. Therefore, the use of the Poisson assumption should bias, if at all, the results of the investigation toward predicting more portfolio autocorrelation than is actually observed.

where $\mathrm{Var}(\tilde{R}_{mt})$ equals the variance of the market index, and the terms θ, δ, γ, and α are functions of the securities' transaction-arrival rates λ_i and coefficients of variation $v_i = \sigma(\hat{R}_{it})/E(\hat{R}_{it})$. For an equal-weighted portfolio, equation (6) becomes

$$\mathrm{Corr}(\tilde{R}_{pt}^T, \tilde{R}_{pt-1}^T) = \frac{[1/n][\overline{\delta_{\lambda_i v_i}\mathrm{Var}(\tilde{R}_{it})}] + [(n-1)/n][\overline{\alpha_{\lambda_i \lambda_j}\beta_i\beta_j}]\mathrm{Var}(\tilde{R}_{mt})}{[1/n][\overline{\theta_{\lambda_i v_i}\mathrm{Var}(\tilde{R}_{it})}] + [(n-1)/n][\overline{\gamma_{\lambda_i \lambda_j}\beta_i\beta_j}]\mathrm{Var}(\tilde{R}_{mt})}, \quad (7)$$

where the bar denotes average.

Scholes and Williams [9] found that for daily data the adjustment factors θ and δ are very close to one and zero, respectively. Consequently, measured variances and autocovariances are very close to true variances and autocovariances. However, γ need not be very close to one, and, therefore, measured contemporaneous covariances may significantly understate true covariances. Substantial first-order cross-covariances also may be induced since α can be significantly different from zero.

The true beta coefficients do not appear to be related to the transaction arrival rates, so that $\alpha_{\lambda_i \lambda_j}$ and $\gamma_{\lambda_i \lambda_j}$ are not related to β_i and β_j.[2] For large n, the value-weighted portfolio autocorrelation is then

$$\mathrm{Corr}(\tilde{R}_{pt}^T, \tilde{R}_{pt-1}^T) = \frac{\sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} x_i x_j \alpha_{\lambda_i \lambda_j}}{\sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} x_i x_j \gamma_{\lambda_i \lambda_j}}. \quad (8)$$

The equal-weighted portfolio autocorrelation for large n reduces to

$$\mathrm{Corr}(\tilde{R}_{pt}^T, \tilde{R}_{pt-1}^T) = [\overline{\alpha_{\lambda_i \lambda_j}}]/[\overline{\gamma_{\lambda_i \lambda_j}}]. \quad (9)$$

Equations (8) and (9) are used in the next section to estimate the theoretical autocorrelation in value- and equal-weighted NYSE indices.

The consequences of the theoretical model for portfolios of different sizes are as follows. For very small n ($n = 1, 2$), the portfolio autocorrelation should be near zero or slightly negative. As n increases, given the sizes of α and γ and reasonable estimates for $\mathrm{Var}(\tilde{R}_{mt})$ and $\mathrm{Var}(\tilde{R}_{it})$, the autocorrelation will be positive and potentially substantially so, depending on the relative sizes of α and γ. The next section details our efforts to estimate the sizes of these latter two terms.

III. Data Description and Implied Theoretical Autocorrelation

The data sample consists of 280 randomly selected firms from the New York Stock Exchange. Daily returns were obtained from CRSP (Center for Research

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[2] The lower transaction firms have a very slight tendency toward smaller-than-average Scholes and Williams' beta estimates. A regression of Scholes and Williams' estimated betas and average daily transactions for the 280 firms had a positive slope coefficient of 0.0002, with a standard error of 0.0005 and a coefficient of determination of only 0.0006.

in Security Prices, University of Chicago) for the 1,011 trading days from January 1978 through December 1981. Continuous returns were calculated as

$$R_{it}^T = \log(1 + r_{it}^T), \tag{10}$$

where r_{it}^T is the daily holding-period return based on the last transaction of the day.

Daily transaction-frequency data for these firms were obtained for the three-month period January through March 1980 from the Francis Emory Fitch Company. The distributions of transaction-arrival rates and market values are needed to evaluate the theoretical portfolio autocorrelation of equations (6) and (7) for large value- and equal-weighted portfolios. A stratified sample of fifty firms from the 280 firms was drawn to reduce the number of α and γ terms from 78,120 to 2,450 in equation (8). The expressions for α and γ (see the Appendix) were calculated using the stratified sample distributions for transactions and market values.

IV. Empirical Results and Analysis

The empirical portfolio autocorrelations, the theoretically predicted portfolio autocorrelations, and a profile of the individual firms is presented in Table I. Clearly, the level of the observed 280-firm portfolio autocorrelation is substan-

Table I
Empirical and Theoretical Results

	Equal-Weighted	Value-Weighted
A. Empirical Portfolio Autocorrelation		
280-Firm NYSE Sample Portfolio	.2586	.1286
NYSE Indices	.2893	.1644*
CRSP Indices	.3130	.1676
B. Theoretical Portfolio Results Due to Nonsynchronous Trading from Equations (8) and (9)		
Average Cross-Covariance Adjustment Factor ($\alpha_{\lambda_i \lambda_j}$)	.0378	.0167
Average Covariance Adjustment Factor ($\gamma_{\lambda_i \lambda_j}$)	.9247	.9659
Portfolio Autocorrelation for Large n	.0408	.0172
C. Individual Security Averages		
Autocovariance	−.00000094	
Cross-Covariance	.00001866	
Variance	.00052890	
Covariance	.00007205	
Autocorrelation	.02451**	
Daily Transaction Frequency	40.66 (Range: 1.88 to 286.08)	

* Daily returns on the value-weighted NYSE Composite Index were calculated from the reported level of the index and do not include dividend returns. All other index returns were calculated from the CRSP daily-returns database and include both capital gain and dividend income.

** Each individual autocorrelation estimate has a standard error of 0.0325.

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tially higher than predicted for both the value- and the equal-weighted portfolios. The autocorrelation of the 280-firm portfolio represents well the autocorrelation level of the CRSP and NYSE portfolios.

Value weighting has a much greater impact on the average cross-covariance adjustment factor than it does on the average covariance adjustment factor. As a result, the portfolio autocorrelation is greatly reduced when market value weights are employed. In fact, the implied theoretical portfolio autocorrelation for the 280-firm portfolio is reduced by approximately sixty percent, while the empirical autocorrelation falls by about fifty percent.

In order to examine the empirical behavior of portfolio autocorrelation as n increases, firms were randomly selected from the sample and entered into a portfolio until all 280 firms were included. The portfolio autocorrelations were recalculated each time a firm was added to the portfolio. This procedure was repeated ten times to obtain ten different realizations of the portfolio size effect on autocorrelation. The ten repetitions were then averaged to capture the portfolio size effect.

Figure 1 compares the empirical autocorrelation pattern in the equal-weighted portfolio with the theoretical pattern due only to nonsynchronous trading effects predicted by equation (7). Note that the high observed autocorrelation of the market index does not depend on there being a very large number of securities. The portfolio-autocorrelation augmentation effect arises almost as rapidly as the portfolio-variance reduction effect associated with the diversification of nonmarket risk.

PORTFOLIO AUTOCORRELATION COMPARISON

EQUAL WEIGHTED DAILY RETURNS
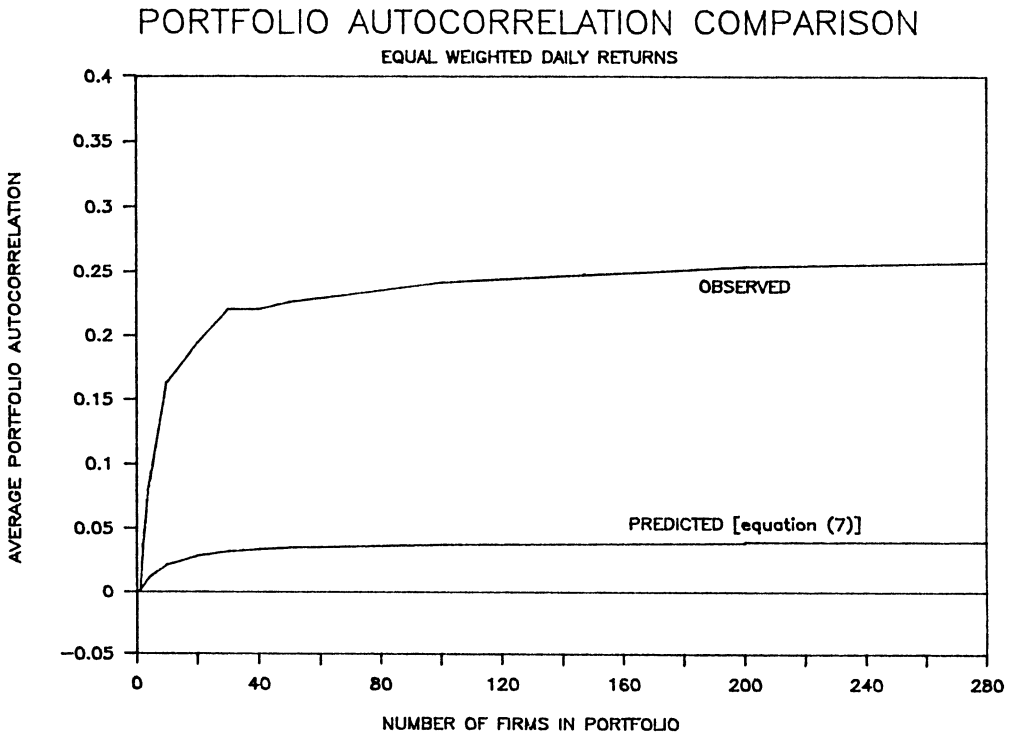
Figure 1. Portfolio Autocorrelation Comparison

V. Conclusions

The Scholes and Williams model of nonsynchronous trading has been combined with empirical data on transaction arrival rates to develop a model of the implied theoretical portfolio autocorrelation. This model is intended to test whether or not the autocorrelation levels observed for the CRSP and NYSE value- and equal-weighted indices are explainable in terms of nonsynchronous trading effects alone.

The level of the autocorrelation predicted from nonsynchronous trading effects was well below that observed. The autocorrelation implied by the theoretical model was only 15.8 percent of that observed for the equal-weighted 280-firm portfolio and 13.4 percent for the value-weighted portfolio. We are led to the conclusion that at present the high observed autocorrelation of the indices examined is not well explained by the nonsynchronous trading model. Other factors appear to be playing the major role in generating the autocorrelations.

Cohen et al. [3, 4] have developed a more complex trading-process model that incorporates some of these frictional, quotation-price-adjustment lags. However, these efforts presently have not resulted in a model formulation that quantifies the index autocorrelation induced from these other frictional sources. This paper demonstrates the need for such a model, which may be more descriptive than the nonsynchronous trading model evaluated here.

Appendix

Scholes and Williams [9] developed the following relationships between the transaction-based return parameters and the terms of the true return variance-covariance matrix:

$$\text{Cov}(\tilde{R}_{it}^T, \tilde{R}_{jt-1}^T) = \alpha_{\lambda_i \lambda_j} \text{Cov}(\tilde{R}_{it}, \tilde{R}_{jt}), \tag{A1}$$

$$\text{Cov}(\tilde{R}_{it}^T, \tilde{R}_{jt}^T) = \gamma_{\lambda_i \lambda_j} \text{Cov}(\tilde{R}_{it}, \tilde{R}_{jt}), \tag{A2}$$

$$\text{Cov}(\tilde{R}_{it}^T, \tilde{R}_{it-1}^T) = \delta_{\lambda_i v_i} \text{Var}(\tilde{R}_{it}), \tag{A3}$$

$$\text{Var}(\tilde{R}_{it}^T) = \theta_{\lambda_i v_i} \text{Var}(\tilde{R}_{it}), \tag{A4}$$

where

$$\alpha_{\lambda_i \lambda_j} = \left[\frac{1}{(1 - e^{-\lambda_i})(1 - e^{-\lambda_j})} \right]$$

$$\times \left[\frac{\lambda_j (1 - e^{-(\lambda_i + \lambda_j)})}{\lambda_i (\lambda_i + \lambda_j)} - e^{-(\lambda_i + \lambda_j)} - [1 + (1/\lambda_i) - (1/\lambda_j)] e^{-\lambda_i} (1 - e^{-\lambda_j}) \right], \tag{A5}$$

$$\gamma_{\lambda_i \lambda_j} = 1 - \alpha_{\lambda_i \lambda_j} - \alpha_{\lambda_j \lambda_i}, \tag{A6}$$

$$\delta_{\lambda_i v_i} = - \left[\frac{1}{\lambda_i^2} + \frac{1}{1 - e^{-\lambda_i}} - \frac{1}{(1 - e^{-\lambda_i})^2} \right] \Big/ v_i^2, \tag{A7}$$

and

$$\theta_{\lambda_i v_i} = 1 - 2\delta_{\lambda_i v_i}. \tag{A8}$$

See Atchison, Butler, and Simonds [1] for tables of the adjustment factors α, γ, δ, and θ for the range of transaction-arrival rates estimated in the study.

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